Static Fields Homework 3

Due 4/11/18 @ 4:00 pm

Start your homework early and submit a question about it on Canvas before class on Tuesday!

Remember that you should do some sense-making about every problem and result (*e.g.,* describe how you know a result is correct, interpret your answer non-symbolically, or describe new physics insight you gained). Solutions that contain exceptional sense-making will receive bonus points.

PRACTICE:

1. When physicists calculate the value of a physical quantity from an equation, they pay particular attention to the units involved. A force of 2 is ill-defined, a force of 2 Newtons is clear. When physicists want to check the plausibility of an equation, without worrying exactly about which set of units will be used (e.g. Newtons vs. pounds vs. dynes), they often look at the "dimensions" of the physical quantities involved. "Dimension" refers to the powers of the basic physical quantities: length (L), time (T), mass (M), and charge (C), that make up the physical quantity. For example, since force is mass times acceleration,the dimensions of force are ML/T^2. Find the dimensions of electrostatic potential energy. Also, find the dimensions of electrostatic potential.

QUIZ:

1. Give the expression for $d\vec{r}$ in rectangular, cylindrical, and spherical coordinates.

REQUIRED:

2. Consider a series of three charges arranged in a line along the z-axis, charges $+Q$ at $z = \pm D$ and charge $-2Q$ at $z = 0$.

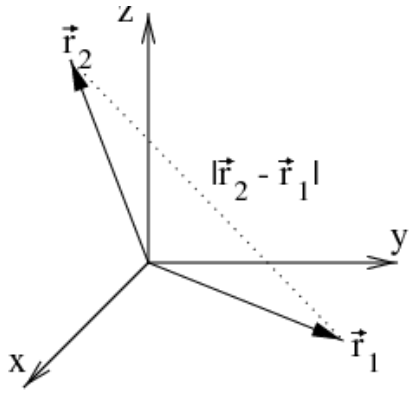
   (a) Find the electrostatic potential at a point P in the xy-plane at a distance r from the center of the quadrupole.

   (b) Assume $r >> D$. Find the first two non-zero terms of a power series expansion to the electrostatic potential you found in the first part of this problem.

   (c) A series of charges arranged in this way is called a linear quadrupole. Why?

3. The distance $|\vec{r} - \vec{r}\,'|$ between the point $\vec{r}\,' = (x', y', z')$ and the point $\vec{r} = (x, y, z)$ is a coordinate-independent, physical and geometric quantity. But, in practice, you will need to know how to express this quantity in different coordinate systems.

   (a) Find the distance $|\vec{r} - \vec{r}\,'|$ between the point $\vec{r}\,' = (x', y', z')$ and the point $\vec{r} = (x, y, z)$ in rectangular coordinates.



(b) Show that this same distance written in cylindrical coordinates is:

$$|\vec{r} - \vec{r}\,'| = \sqrt{s^2 + s'^2 - 2ss'\cos(\phi' - \phi) + (z' - z)^2}$$

(c) Show that this same distance written in spherical coordinates is:

$$|\vec{r}\,' - \vec{r}| = \sqrt{r'^2 + r^2 - 2rr'\left[\sin\theta\sin\theta'\cos(\phi' - \phi) + \cos\theta'\cos\theta\right]}$$

(d) Now assume that $\vec{r}\,'$ and $\vec{r}$ are in the x-y plane. Simplify the previous two formulas.

4. A helix with 17 turns has height H and radius R. Charge is distributed on the helix so that the charge density increases like the square of the distance up the helix. At the bottom of the helix the linear charge density is 0 $\frac{\mathrm{C}}{\mathrm{m}}$. At the top of the helix, the linear charge density is 13 $\frac{\mathrm{C}}{\mathrm{m}}$. What is the total charge on the helix?