**Static Fields Homework 2**

*Due 4/6/18 @ 4:00 pm*

Start your homework early and submit a question about it on Canvas before class on Thursday!

Remember that you should do some sense-making about every problem and result (*e.g.,* describe how you know a result is correct, interpret your answer non-symbolically, or describe new physics insight you gained). Solutions that contain exceptional sense-making will receive bonus points.

## PRACTICE:

1. Write out the first four nonzero terms in the series:

   (a)
   $$\sum_{n=0}^{\infty} \frac{1}{n!}$$

   (b)
   $$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$$

2. Write the following series using sigma ($\sum$) notation.

   (a)
   $$1 - 2\,\theta^2 + 4\,\theta^4 - 8\,\theta^6 + \ \ldots$$

   (b)
   $$\frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \frac{1}{25} + \ \ldots$$

3. If you need more practice with sigma ($\sum$) notation, you can get really good practice by going back and forth between the two representations of the standard power series on the memorization page. Power series are used everywhere in physics and it is very important to be able to translate back and forth between the two representations.

## QUIZ:

1. Be able to give the first four nonzero terms of the power series for $\sin z$, $\cos z$, $e^z$, $\ln(1+z)$, and $(1+z)^p$.

## REQUIRED:

2. Consider the geometry of $|\vec{\boldsymbol{r}} - \vec{\boldsymbol{r}}'|$.

(a) Make sketches of the following functions, by hand, all on the same axes. Briefly describe, using good scientific writing that includes both words and equations, the role that the number 2 plays in the shape of the second graph:

$$y = \sin x \tag{1}$$
$$y = \sin(2+x) \tag{2}$$

(b) Make a sketch of the graph

$$|\vec{r} - \vec{a}| = 2$$

for each of the following values of $\vec{a}$:

$$\vec{a} = \vec{0} \tag{3}$$
$$\vec{a} = 2\hat{\imath} - 3\hat{\jmath} \tag{4}$$
$$\vec{a} = \text{points due east and is 2 units long} \tag{5}$$

(c) Derive a more familiar equation equivalent to

$$|\vec{r} - \vec{a}| = 2$$

for arbitrary $\vec{a}$, by expanding $\vec{r}$ and $\vec{a}$ in rectangular coordinates. Simplify as much as possible. (Ok, ok, I know this is a terribly worded question. What do I mean by "more familiar"? What do I mean by "simplify as much as possible"? Why am I making you read my mind? Try it anyway. Real life is not full of carefully worded problems. Bonus points to anyone who can figure out a better way of wording the question that doesn't give the point away.)

(d) Write a brief description of the geometric meaning of the equation

$$|\vec{r} - \vec{a}| = 2$$

3. Using a dot product, find the angle between any two line segments that join the center of a regular tetrahedron to its vertices. *Hint:* Think of the vertices of the tetrahedron as sitting at the vertices of a cube (at coordinates (0,0,0), (1,1,0), (1,0,1) and (0,1,1)—you may need to build a model and play with it to see how this works!)

4. The change in gravitational potential energy can be found by integrating

$$\Delta U = \int_{r_i}^{r_f} G\frac{m_1 m_2}{r^2} dr$$

where $G$ is the gravitational constant, $m_1$ and $m_2$ are masses, and $r$ is the distance between the two masses.

(a) Perform the integration, showing all steps.

(b) Plot the potential energy, $\Delta U$, as the mass, $m_1$, varies. Label significant points on the plot and describe (in words) the behavior.

(c) Plot the potential energy, $\Delta U$, as the final distance, $r_f$, varies. Label significant points on the plot and describe the behavior.

5. Recall that, if you take an infinite number of terms, the series for $\sin z$ and the function itself $f(z) = \sin z$ are equivalent representations of the same thing for all real numbers $z$, (in fact, for all complex numbers $z$). This is not always true. More commonly, a series is only a valid, equivalent representation of a function for some more restricted values of $z$. The technical name for this idea is convergence–the series only "converges" to the value of the function on some restricted domain.

   Find the power series for the function $f(z) = \frac{1}{1+z^2}$. Then, using the Mathematica worksheet from class (vfpowerapprox.nb) as a model, explore the convergence of this series. Where does your series for this new function converge? Can you tell anything about the region of convergence from the graphs of the various approximations? Print out a plot and write a brief description (a sentence or two) of the region of convergence.

   Note: As a matter of professional ettiquette (or in some cases, as a legal copyright requirement), if you use or modify a computer program written by someone else, you should always acknowledge that fact briefly in whatever you write up. Say something like: "This calculation was based on a (*name of software package*) program titled (*title*) originally written by (*author*) copyright (*copyright date*).