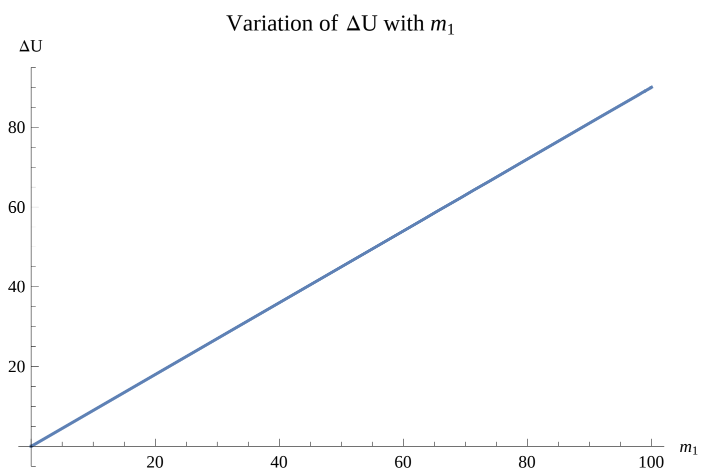
4

Part B.

$$\Delta U = G * m1 * m2 \left(\frac{1}{ri} - \frac{1}{rf}\right)$$

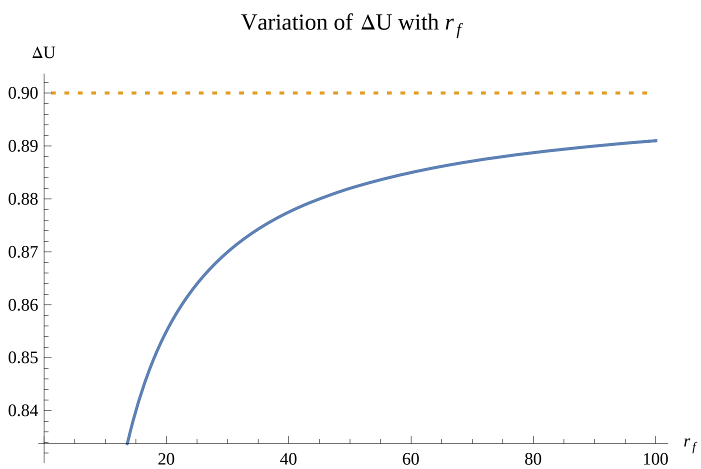
```
Clear[m1]
rf = 10
ri = 1
G = 1
m2 = 1
Plot[ΔU, {m1, 0, 100 m2}, AxesLabel → {"m₁", "ΔU"}, PlotLabel → "Variation of ΔU with m₁"]
```



As we can see, the relationship between ΔU and m_1 is linear. It is evident from the formula: $\Delta U = G\, m_1\, m_2 \left(\frac{1}{r_i} - \frac{1}{r_f}\right)$ as if we fix m_2, r_i and r_f, ΔU varies linearly with m_1. The slope of the curve will be positive (and constant) when rf is chosen to be less than ri, and negative and constant otherwise. We should not plot negative values as we do not have the concept of a negative mass.

■ Part C.

```
Clear[rf]
m1 = 0.9
Plot[{ΔU, G m1 m2/ri}, {rf, 1.1, 100}, AxesLabel → {"r_f", "ΔU"},
 PlotLabel → "Variation of ΔU with r_f", PlotStyle → {Dashing[None], Dashing[Small]}]
Limit[ΔU, rf → ∞]
```




```
Clear[rf, ri, m1, m2, G]
Limit[ΔU, rf → ∞]
```

$$\frac{G\, m1\, m2}{ri}$$

The blue curve represents the relationship between ΔU and r_f. The slope is positive at all points. The curve has an asymptotic behavior and approaches the value $G \frac{m_1 m_2}{r_i}$ (which is the limit as $r_f \to \infty$) shown in orange.