Problem 3





Sense-making: The syntax for plotting in 2D and 3D is basically the same! Also, it's worthwhile to note that you can have a function of two variables that is constant for one (or both) of them, which is cool.

Problem 4

```
V[x_{, y_{-}}] = 1 / Sqrt[x^{2} + y^{2}];
ContourPlot[V[x, y], {x, -10, 10}, {y, -10, 10}, Contours \rightarrow 10]
```



Sense-making: Some things I have noticed about the graph are that the circles are getting farther and farther apart as the value of r increases, except the middle region is all filled. *Mathematica* has chosen to not include contours in that

region close to the origin because the values grow too steeply, and to show them would cause loss of resolution across the rest of the plot. A surface plot showing the same cutoff is below.



Problem 5

Integral 1

```
Integrate [Sin[x^2], {x, 0, Sqrt[2Pi]}]
N[Integrate[Sin[x^2], {x, 0, Sqrt[2Pi]}]]
\sqrt{\frac{\pi}{2}} FresnelS[2]
0.430408
```

```
Sense-making: Since sine is bounded by +/- 1, I definitely expected this integral to be bounded by +/- \sqrt{2\pi} and it certainly is. The Fresnel function isn't necessarily helpful here, so we can find the approximate numerical value, as this is a definite integral, and plot the integrand and look at it in the region of interest:
```



Okay, so it looks like there is more positive area than negative area in this region, but that they magnitudes are pretty close. So a small number like 0.43 makes sense, and seems to be on the right order of magnitude.

Integral 2

```
\label{eq:integrate} \begin{split} \texttt{Integrate} \Big[\texttt{Sin}[\texttt{x}\,\texttt{y}]\,,\, \Big\{\texttt{y},\,\texttt{0}\,,\,\texttt{1}\,-\,\texttt{x}^2\Big\}\Big]\,,\, \{\texttt{x},\,-\,\texttt{1}\,,\,\texttt{1}\,\}\Big] \end{split}
```

0

Sense-making: This result of zero immediately makes me suspicious. But if there is something symmetric happening, the result is more likely. Let's do part of the math so we can plot stuff:

At this point it is almost obvious that since we are integrating the above (antisymmetric) function over symmetric bounds, the result should be zero.

Alternate sense-making: The limits of integration are unusual here--they correspond to a region in the xy-plane

bounded by the x-axis and by an inverted parabola. A surface plot and a contour plot of the integrand are shown over only the region of integration:



Aha, Mathematica can do integrals involving arbitrary parameters. This bodes well for lazy physicists.

Integral 4

Integrate[1 / (Sqrt[1 - A Cos[x]]), {x, 0, 2 Pi}]

$$\label{eq:conditionalExpression} \begin{split} & \text{ConditionalExpression} \Big[\frac{4 \, \text{EllipticK} \big[\frac{2 \, \text{A}}{1 + \text{A}} \big]}{\sqrt{1 + \text{A}}} \, \text{, } \text{Im}[\text{A}] \neq 0 \, \mid \mid -1 \leq \text{Re}[\text{A}] \leq 1 \Big] \end{split}$$

Wow! *Mathematica* looks angry! My guess is that it has some issues with the possibility of complex-valued functions. I will try to remedy this:

 $\label{eq:linear} Integrate[1 / (Sqrt[1 - A Cos[x]]), \{x, 0, 2 Pi\}, Assumptions \rightarrow Abs[A] < 1]$

$$\frac{4 \text{ EllipticK}\left[\frac{2 \text{ A}}{1 + \text{A}}\right]}{\sqrt{1 + \text{A}}}$$

I guess that is as simplified as it will get without choosing values for A. It would porbably be a good idea to look up EllipticF and see what that function does...

Anyhow, I can do some manipulation of the function I want to integrate to see what happens for different values of A (But it won't show up in a PDF, so you can copy and play with it yourself : Manipulate[Plot[1/(Sqrt[1-A Cos[x]]), {x,-10,10}], {A,-2,2}]

Let's see what else we can do:

```
\texttt{Plot[NIntegrate[1 / (Sqrt[1 - A Cos[x]]), \{x, 0, 2 Pi\}], \{A, -1, 1\}, AxesLabel \rightarrow \{A\}]}
```



Above is a plot of the value of I_4 as a function of A. It really only makes sense to consider values of A between -1 and 1, as the square root gives a complex result (unless, of course, you are solving a problem where a complex number makes sense as the answer ...).