8.4 #22 $\quad w' + xw = e^x, \quad w = \sum_{n=0}^{\infty} a_n x^n$

$$\sum_{n=0}^{\infty} n a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^{n+1} = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

or $(a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \dots) + (a_0 x + a_1 x^2 + a_2 x^3 + \dots) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots$

$\Rightarrow a_1 = 1$

$2a_2 + a_0 = 1 \to a_2 = \frac{1}{2}(1 - a_0)$

$3a_3 + a_1 = \frac{1}{2} \to a_3 = \frac{1}{3}\left(\frac{1}{2} - 1\right) = -\frac{1}{6}$

$4a_4 + a_2 = \frac{1}{6} \to a_4 = \frac{1}{4}\left(\frac{1}{6} - a_2\right) = \frac{1}{4}\left(\frac{1}{6} - \frac{1}{2} + \frac{1}{2}a_0\right) = \frac{1}{4}\left(-\frac{2}{6} + \frac{1}{2}a_0\right) = \frac{1}{8}\left(-\frac{2}{3} + a_0\right)$

So $w(x) = a_0\left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 + \dots\right) + \left(x + \frac{1}{2}x^2 - \frac{1}{6}x^3 - \frac{1}{12}x^4 + \dots\right)$

8.6 #32 $\quad x^2 y'' - x(1+x)y' + y = 0$. $x = 0$ is reg. sing. pt

$r(r-1) + (-1)r + 1 = 0 \to r^2 - 2r + 1 = 0 \to (r-1)^2 = 0$

$y = x\sum_{n=0}^{\infty} a_n x^n, \; a_0 \neq 0, \quad y' = \sum_{n=0}^{\infty} (n+1) a_n x^n, \quad y'' = \sum_{n=0}^{\infty} (n+1) n a_n x^{n-1}$

$$\to \sum_{n=0}^{\infty} (n+1) n a_n x^{n+1} - \sum_{n=0}^{\infty} (n+1) a_n x^{n+1} - \underbrace{\sum_{n=0}^{\infty} (n+1) a_n x^{n+2}}_{n' = n+1} + \sum_{n=0}^{\infty} a_n x^{n+1} = 0$$

$$\sum_{n'=1}^{\infty} n' a_{n'-1} x^{n'+1}$$

$$= \sum_{n=0}^{\infty} n a_{n-1} x^{n+1}$$

$$\to \sum_{n=0}^{\infty} \left\{ \Big[\underbrace{(n+1)n - (n+1) + 1}_{n^2 \cancel{+n} \cancel{\pm 1}} \Big] a_n - n a_{n-1} \right\} x^{n+1} = 0 \Rightarrow \sum_{n=0} \left[n^2 a_n - n a_{n-1} \right] x^{n+1} = 0$$

$n = 0$ always ok

$\Rightarrow n^2 a_n - n a_{n-1} = 0, \; n = 0, 1, 2, \dots \to a_n = \frac{1}{n} a_{n-1}, \; n = 1, 2, \dots$

$\Rightarrow$ If $a_0 < 0$, then $a_n < 0$ for all $n = 1, 2, \dots$

So then $y = \sum_{n=0}^{\infty} a_n x^n$ and $y' = \sum_{n=0}^{\infty} n a_n x^{n-1} < 0$

for $x > 0$, hence

y is decreasing with this choice of