12.5 #15

$$\begin{cases} \dot{x} = -y + x f(x,y) \\ \dot{y} = x + y f(x,y) \end{cases}$$

(a) $(0,0)$ is asy stable if $f(x,y) < 0$ in D

(b) — unstable if $f(x,y) > 0$ in D

(a) $V = \frac{1}{2}x^2 + \frac{1}{2}y^2 > 0$ if $(x,y) \neq (0,0)$ and smooth everywhere

$$\dot{V} = x(-y + xf) + y(x + yf) = (x^2+y^2) f < 0 \text{ in } D$$

$\Rightarrow (0,0)$ asy stable by L's stab Thm.

(b) $V = \frac{1}{2}x^2 + \frac{1}{2}y^2$ so $V(0,0) = 0$

$\dot{V} = (x^2+y^2) f > 0$ in D

For all $R > 0$, there's p in $B_R(0)$: $V(p) > 0$

$\Rightarrow (0,0)$ unstable by Chetaev's Thm (called L's instab Thm in te

12.6 #8

$$\begin{cases} \dot{x} = x - y + x(x^2+y^2) \\ \dot{y} = x + y + y(x^2+y^2) \end{cases}$$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \quad \begin{cases} \dot{x} = r\cos\theta - r\sin\theta + r^3\cos\theta = \dot{r}\cos\theta - r\sin\theta \\ \dot{y} = r\cos\theta + r\sin\theta + r^3\sin\theta = \dot{r}\sin\theta + r\cos\theta \end{cases}$$

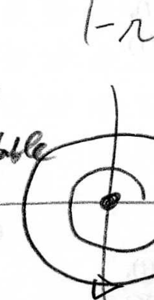
$$\Rightarrow \begin{cases} r + r^3 = \dot{r} \\ -r = -r\dot{\theta} \end{cases} \quad \text{or} \quad \begin{cases} \dot{r} = r(r^2+1) \\ (r \neq 0) \ \dot{\theta} = +1 \end{cases}$$

counterclockw

clearly $\dot{r} = 0 \Leftrightarrow r = 0$ and $\dot{r} > 0$ if $r > 0$

Thus only 1 critical point at origin which is unstable

No nontrivial periodic solutions



12.6 #16

$$\begin{cases} \dot{x} = 3x + 2y - xy^2 \\ \dot{y} = x + 4y - 2x^2 y \end{cases}$$

has no nonconstant periodic solutions in interior of ellipse $2x^2 + y^2 = 7$

$$\frac{\partial}{\partial x}(3x + 2y - xy^2) + \frac{\partial}{\partial y}(x + 4y - 2x^2y) = 3 - y^2 + 4 - 2x^2 = 7 - y^2 - 2x^2 > 0 \text{ if}$$

(x,y) is in terior of ellips $2x^2+y^2=7$

By Bendixon's criterion, there are no nonconstant periodic solutions in interior of ellipse $2x^2 + y^2 = 7$.