

12.2 #10 classify eqpt

$$\begin{cases} \dot{x} = 2x + 4y + 4 \\ \dot{y} = 3x + 5y + 4 \end{cases}$$

$$\det \begin{pmatrix} 2-\lambda & 4 \\ 3 & 5-\lambda \end{pmatrix} = \lambda^2 - 7\lambda - 2$$

$$\lambda = \frac{7 \pm \sqrt{49+8}}{2} \quad \begin{matrix} \text{one negative} \\ \text{one positive} \end{matrix}$$

⇒ Saddle, unstable

12.3 #12 Find eq., classify + discuss stab.

$$\begin{cases} \dot{x} = 4 - xy \\ \dot{y} = x - y \end{cases}$$

$$x=y \text{ and } 4-x^2=0 \text{ ie } (2,2) \text{ and } (-2,-2)$$

$$\text{Jac} = \begin{pmatrix} -y & -x \\ 1 & -1 \end{pmatrix}$$

$$\begin{matrix} \begin{pmatrix} -2 & -2 \\ 1 & -1 \end{pmatrix} & \begin{pmatrix} 2 & 2 \\ 1 & -1 \end{pmatrix} \end{matrix}$$

$$\begin{matrix} \text{tr} = -3 \\ \det = 4 \end{matrix} \Rightarrow \text{a.s.} \\ \text{tr}^2 - 4\det = 9 - 16 < 0 \Rightarrow \text{stable spiral}$$

$$\begin{matrix} \text{tr} = 1 \\ \det = -4 \end{matrix} \begin{matrix} \text{Saddle} \\ \text{Unstab} \end{matrix}$$

12.4 #10

$$\frac{d^2x}{dt^2} - \sin x = 0, \quad q(x) = -\sin x$$

$$b(x) = \int q = \cos x + C \quad (\text{potential})$$

$$V = \frac{dx}{dt} \Rightarrow E(x,v) = \frac{1}{2}v^2 + b = \frac{1}{2}v^2 + \cos x + C, \quad E(0,0) = 0 \Rightarrow C = -1$$

$$b(x) = \cos x - 1$$

