9.8 #12

$$A=\begin{bmatrix}1&1&1\\2&1&-1\\0&-1&1\end{bmatrix} \qquad \det\begin{bmatrix}1-\lambda&1&1\\2&1-\lambda&-1\\0&-1&1-\lambda\end{bmatrix}=(1-\lambda)\left[(1-\lambda)^2-1\right]-2\left[1-\lambda+1\right]$$

$$=(1-\lambda)\left[\lambda^2-2\lambda+1-1\right]-2\left[2-\lambda\right]=\lambda(1-\lambda)\left[\lambda-2\right]+2\left[\lambda-2\right]=(\lambda-2)\left[-\lambda^2+\lambda+2\right]$$

$$\lambda=\frac{-1\pm\sqrt{1+8}}{-2}=\frac{-1\pm3}{-2}=$$

$$\lambda=-1 \quad \left[\begin{array}{ccc|c}2&1&1&0\\2&2&-1&0\\0&-1&2&0\end{array}\right]\to\left[\begin{array}{ccc|c}2&1&1&0\\0&1&-2&0\\0&-1&2&0\end{array}\right]\to\left[\begin{array}{ccc|c}2&1&1&0\\0&1&-2&0\\0&0&0&0\end{array}\right]\to\left[\begin{array}{ccc|c}1&0&3/2&0\\0&1&-2&0\\0&0&0&0\end{array}\right]$$

$$\begin{bmatrix}-\frac{3}{2}\\2\\2\end{bmatrix} \text{ or } \begin{bmatrix}-3\\4\\2\end{bmatrix}$$

$$\lambda=2 \quad \begin{bmatrix}-1&1&1\\2&-1&-1\\0&-1&-1\end{bmatrix}\begin{bmatrix}-1&1&1\\2&-1&-1\\0&-1&-1\end{bmatrix}=\begin{bmatrix}3&-3&-3\\-4&4&4\\-2&2&2\end{bmatrix} \text{ or } \begin{array}{c}x-y-z=0\\ y\begin{bmatrix}1\\1\\0\end{bmatrix}+z\begin{bmatrix}1\\0\\1\end{bmatrix}\end{array}$$

$$(A-2I)\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}0\\+1\\-1\end{bmatrix} \quad \text{but } (A-2I)\begin{bmatrix}0\\+1\\-1\end{bmatrix}=0$$

$$x_3(t)=e^{2t}\left[u+t(A-2I)u\right]=e^{2t}\begin{bmatrix}1\\1+t\\0-t\end{bmatrix}$$

$$X(t)=\begin{bmatrix}-3e^{-t}&0&e^{2t}\\4e^{-t}&e^{2t}&e^{2t}(1+t)\\2e^{-t}&-e^{2t}&-te^{2t}\end{bmatrix} \qquad X(0)=\begin{bmatrix}-3&0&1\\4&1&1\\2&-1&0\end{bmatrix}$$

$$X^{-1}(0):\left[\begin{array}{ccc|ccc}-3&0&1&1&0&0\\4&1&1&0&1&0\\2&-1&0&0&0&1\end{array}\right]\to\left[\begin{array}{ccc|ccc}+1&0&-\frac{1}{3}&-\frac{1}{3}&0&0\\0&1&7/3&4/3&1&0\\0&-1&2/3&2/3&0&1\end{array}\right]\to\left[\begin{array}{ccc|ccc}1&0&-\frac{1}{3}&-\frac{1}{3}&0&\\0&1&7/3&4/3&1&\\0&0&3&2&1&1\end{array}\right]$$

$$\to\left[\begin{array}{ccc|ccc}1&0&0&-\frac{1}{3}+\frac{2}{9}&\frac{1}{9}&\frac{1}{9}\\0&1&0&4/3-\frac{14}{9}&1-\frac{7}{9}&-\frac{7}{9}\\0&0&1&\frac{2}{3}&\frac{1}{3}&\frac{1}{3}\end{array}\right] \text{ or } X^{-1}(0)=\begin{bmatrix}-1/9&1/9&1/9\\-2/9&2/9&-7/9\\2/3&1/3&1/3\end{bmatrix}=$$

$$X_0^{-1}=\frac{1}{9}\begin{bmatrix}-1&1&1\\-2&2&-7\\6&3&3\end{bmatrix}$$