9.4 #26

$x_1\begin{bmatrix} e^{3t} \\ 0 \\ e^{3t}\end{bmatrix}$ $x_2\begin{bmatrix} -e^{3t} \\ e^{3t} \\ 0\end{bmatrix}$ $x_3\begin{bmatrix} -e^{-3t} \\ -e^{-3t} \\ -e^{-3t}\end{bmatrix}$ solve $\dot{x}=\underbrace{\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ 2 & -2 & 1\end{bmatrix}}_{A}x$

$W[x_1, x_2, x_3] \neq 0$ for all t

trivial; plug into equa

$x_p=\begin{bmatrix} 5t+1 \\ 2t \\ 4t+2\end{bmatrix}$ is part. sol. to $\dot{x}=Ax+\begin{pmatrix} -9t \\ 0 \\ -18t\end{pmatrix}$

Gen Sol: $X(t)=\begin{bmatrix} x_1 & x_2 & x_3\end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3\end{bmatrix}+x_p$

9.4 #33

Use Abel to prove that $w(t)=0$ for all t or $\neq 0$ for any t

Abel

$W(t_0)\neq 0 \Rightarrow W(t)\neq 0$ for a

$W(t_0)=0 \Rightarrow W(t)=0$ for a

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(a)

(b) $x_1(t)=\begin{bmatrix} \sin t \\ \sin t \\ 0\end{bmatrix}$, $x_2(t)=\begin{bmatrix} \sin t \\ 0 \\ \sin t\end{bmatrix}$, $x_3(t)=\begin{bmatrix} 0 \\ \sin t \\ \sin t\end{bmatrix}$ lin. indep

Pick $t=\frac{\pi}{2}$: $\det\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1\end{bmatrix}=-2\neq 0$

(c) $W=\sin^3 t\cdot(-2)$

(d) Is there a lin sys.? No since W sometimes 0, sometimes $\neq 0$

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$x'=\begin{bmatrix} 1 & 3 & -2 \\ 0 & 7 & -4 \\ 0 & 9 & -5\end{bmatrix}$ $\det\begin{bmatrix} 1-\lambda & 3 & -2 \\ 0 & 7-\lambda & -4 \\ 0 & 9 & -5-\lambda\end{bmatrix}=(1-\lambda)\left[(\lambda-7)(\lambda+5)+36\right]$

$=(1-\lambda)(\lambda^2-2\lambda+1)=(\lambda-1)^3$

$\left[\begin{array}{ccc|c} 0 & 3 & -2 & 0 \\ 0 & 6 & -4 & 0 \\ 0 & 9 & -6 & 0\end{array}\right] \to \left[\begin{array}{ccc|c} 0 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{array}\right]$ $\Big\}$ $x\begin{pmatrix} 1 \\ 0 \\ 0\end{pmatrix}+y\begin{pmatrix} 0 \\ 2 \\ 3\end{pmatrix}$; $x+y\neq 0$

$x_1(t)=e^{\lambda t}\begin{pmatrix} 1 \\ 0 \\ 0\end{pmatrix}$, $x_2(t)=e^{\lambda t}\begin{pmatrix} 0 \\ 2 \\ 3\end{pmatrix}$

$(A-I)u_4=(u_3) \to \left[\begin{array}{ccc|c} 0 & 3 & -2 & 1 \\ 0 & 6 & -4 & 2 \\ 0 & 9 & -6 & 3\end{array}\right] \to \left[\begin{array}{ccc|c} 0 & 3 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{array}\right] \to 3y-2z=1 \Rightarrow$ $x=0$, $y=0$, $z=-\frac{1}{2}$

pick $\begin{pmatrix} 1 \\ 0 \\ 0\end{pmatrix}+\begin{pmatrix} 0 \\ 2 \\ 3\end{pmatrix}$

$x_3(t)=e^{\lambda t}\left(t\begin{pmatrix} 1 \\ 2 \\ 3\end{pmatrix}+\begin{pmatrix} 0 \\ 0 \\ -\frac{1}{2}\end{pmatrix}\right)$