Exam 3: MAP 4015*

December 5, 2012

Name:

This is a **closed book** exam and the use of formula sheets or calculators is **not** allowed.

1. Let
$$A = \begin{pmatrix} 1 & a \\ a & 1 \end{pmatrix}$$
where a is a real parameter. Find a closed-form expression for A^n, where n is an arbitrary positive integer.

2. Let $A \in M_{n\times n}(\mathbb{R})$ be an invertible matrix with characteristic polynomial $f(t) = (-1)^n t^n + a_{n-1}t^{n-1} + \cdots + a_1 t + a_0$. Show that $a_0 \neq 0$, and that
$$A^{-1} = -\frac{1}{a_0}\left((-1)^n A^{n-1} + a_{n-1}A^{n-2} + \cdots + a_1 I_n\right)$$

3. A matrix $A \in M_{n\times n}(\mathbb{R})$ is said to be orthogonal if:
$$AA^t = I_n$$
Show that if A is orthogonal, then $\det(A)$ can only take two possible values. What are these values?

4. Let $T : V \to V$ be a linear operator and $\dim(V) < \infty$. Suppose that W is a T-invariant subspace of V. Prove that the characteristic polynomial of T_W divides the characteristic polynomial of T.

   [*Recall that T_W is the restriction of T to W, and thus that $T_W : W \to W$ is a linear operator on W.*]

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