

Exam 2: MAP 4015*

October 26, 2012

Name:

This is a **closed book** exam and the use of formula sheets or calculators is **not** allowed.

1. Let β be an ordered basis of a finite-dimensional vector space V , and let $T : V \rightarrow W$ be an isomorphism. Prove that $T(\beta)$ is a basis of W .
2. (i) Let $T : V \rightarrow W$ and $U : W \rightarrow Z$ be linear transformations on finite dimensional vectorspaces V and W respectively. Prove that

$$\text{rank}(UT) \leq \text{rank}(U).$$

- (ii) Let $A \in M_{m \times n}(F)$ and $B \in M_{n \times p}(F)$. Use (i) to prove that:

$$\text{rank}(AB) \leq \text{rank}(A).$$

3. Compute the rank of the following matrix, and determine its inverse-if the inverse exists:

$$A = \begin{pmatrix} 0 & -2 & 4 \\ 1 & 1 & -1 \\ 2 & 4 & -5 \end{pmatrix}$$

4. (i) Use Gaussian elimination to find the reduced row echelon form of the following matrix:

$$A = \begin{pmatrix} 2 & 0 & 3 & 0 & -4 \\ 3 & -4 & 8 & 3 & 0 \\ 1 & -1 & 2 & 1 & -1 \\ -2 & 5 & -9 & -3 & -5 \end{pmatrix}$$

- (ii) Use the result from part (i) to find a basis for $R(L_A)$, the range of L_A .

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