Exam 1: MAP 4015*

September 21, 2012

Name:

This is a **closed book** exam and the use of formula sheets or calculators is **not** allowed.

1. Let V be a vector space of dimension n. Prove that any linearly independent subset of V that contains exactly n vectors, is a basis for V. (Note: Make sure to precisely state, but not prove, any auxiliary result you use to prove this result.)

2. Let S be a subset of $M_{n\times n}(\mathbb{R})$ consisting of the so-called tri-banded matrices, defined as follows:

$$S = \{A \in M_{n\times n}(\mathbb{R}) \mid A_{ij} = 0 \text{ if } |i-j| > 1 \text{ for all } 1 \leq i \leq n \text{ and } 1 \leq j \leq n\}$$

   Prove that S is a subspace of $M_{n\times n}(\mathbb{R})$ and find a basis for S. What is the dimension of S?

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$. Find a formula for $T(a,b)$ where T represents the projection of the point with coordinates (a,b) on the y-axis along the line $L = \{(s,s)|s \in \mathbb{R}\}$. Include a figure of this projection, clearly showing the points (a,b) and $T(a,b)$, and the line L.

   Show that T is linear.

   Determine $N(T)$ and $R(T)$.

   State and verify the Dimension Theorem for T.

   Is T onto? Is it 1-to-1?

4. Show that a subset W of a vector space V is a subspace of V if and only if $\text{span}(W) = W$.

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