# Homework assignment 3*

**Due date: Monday April 13, 2009.**

1. In class we solved McKendrick's problem:

$$u_a(a,t) + u_t(a,t) = -d(a)u(a,t),$$

with

$$BC:\ u(0,t) = B(t),\ \text{and}\ IC:\ u(a,0) = u_0(a).$$

Here we consider the special case where the survival function is exponential:

$$l(a) = \mathrm{e}^{-\mu a},\ \text{for some}\ \mu > 0.$$

- Find the instantaneous death rate $d(a)$.
- Find the basic reproduction number $R_0$ (i.e. the expected number of offspring per female over her lifetime) if the maternity function is given by:

$$m(a) = \begin{cases} 0,\ a \in [0, \mu/2] \\ m,\ a \in (\mu/2, 3\mu/2) \\ 0,\ a > 3\mu/2 \end{cases}$$

- Assuming that the age distribution is such that for all $t > 0$:

$$\lim_{a\to\infty} u(a,t) = 0,$$

show that the total population

$$N(t) = \int_0^\infty u(a,t)da$$

satisfies the linear ODE:

$$\dot{N}(t) = B(t) - \mu N(t).$$

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