HW6 Solutions (Sec 3.5)

21. Sum of 4 dice, Σ_4, belongs to $\{4, 5, \ldots, 24\}$

By symmetry: Let k_i in $\{1,\ldots,6\}$ be face of i^{th} die.

Then if $k_1+k_2+k_3+k_4=k$, also $(7-k_1)+(7-k_2)+(7-k_3)+(7-k_4)=28-k$

and therefore $P(\Sigma_4=k)=P(\Sigma_4=28-k)$

$\to$ Suffices to calculate $P(\Sigma_4=k)$ for $k=4,\ldots,14$.

Now, for any k in $\{4,\ldots,24\}$:

$$P(\Sigma_4=k)=\sum_{i=1}^{6} P(\Sigma_4=k \mid \Sigma_3=k-i)P(\Sigma_3=k-i)=\frac{1}{6}\sum_{i=1}^{6}\boxed{P(\Sigma_3=k-i)}$$

These probabilities are in the table of Ex. 3.15 (p90)

$k=$	4,24	5,23	6,22	...	13,15	14,14
$P(\Sigma_4=k)$						

$$\to P(\Sigma_4=14)=\frac{1}{6}\cdot\frac{1}{216}(21+25+27+27+25+21)$$

$$=\frac{146}{1296}$$, this is maximal probability

34. CB = "person is colorblind", M = "person is male", F = "person is female"

$P(CB|M)=0.05$
$P(CB|F)=0.0025$
$P(M)=P(F)=0.5$

$$\to P(M|CB)=\frac{P(M\cap CB)}{P(CB)}=\frac{P(CB|M)P(M)}{P(CB|M)P(M)+P(CB|F)P(F)}$$

$$=\frac{(0.05)(0.5)}{(0.05)(0.5)+(0.0025)(0.5)}=\frac{20}{21}$$

44. L = "person is liberal", S = "person smokes"
C = "person is conservative"
$P(L)=0.4$, $P(C)=0.6$ (So $L\cup C$ partitions set of all people)
$P(S|L)=0.25$, $P(S|C)=0.5$

$$P(L|S)=\frac{P(S|L)P(L)}{P(S|L)P(L)+P(S|C)P(C)}=\frac{1}{4}$$

47. A = "knows answer", A^c = "guesses"
R = "gets answer right"

$P(A)=0.6$, $P(A^c)=0.4$
$P(R|A)=1$, $P(R|A^c)=0.25$

$$P(A^c|R)=\frac{P(R|A^c)P(A^c)}{P(R|A^c)P(A^c)+P(R|A)P(A)}=\frac{1}{7}$$

55. L = "liberal", C = "conservative", I = "independent"
V = "voted". $P(C)=0.3$, $P(L)=0.5$, $P(I)=0.2$
$P(V|C)=\frac{2}{3}$, $P(V|L)=0.8$, $P(V|I)=0.5$

$$P(L|V)=\frac{P(V|L)P(L)}{P(V|L)P(L)+P(V|C)P(C)+P(V|I)P(I)}=\frac{4}{7}$$