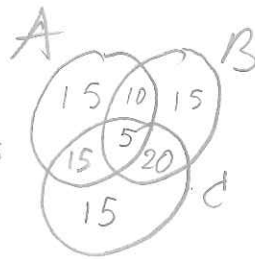


Solutions HW 5



2.7 #80

$A = \text{"boys getting drum"} , |A| = 45$
 $B = \text{"car"} , |B| = 50$
 $C = \text{"bat"} , |C| = 55$
 $|A \cap B| = 15$
 $|A \cap C| = 20$
 $|A \cap B \cap C| = 5$
 $|B \cap C| = 25$

Implicit assumption: All 150 (= 45 + 50 + 55) presents will be distributed.

(a) # Boys getting presents = $|A \cup B \cup C| = 3 \cdot 45 + (10 + 15 + 20) + 5 = 95$

(b) # Boys getting only a drum = $|A \cap (B \cup C)^c| = 15$

2.7 #82 $A_i = \text{"no job on day } i"$, $i = 1 \dots 5$

$$P(\text{"at least 1 day without a job"}) = P\left(\bigcup_{i=1}^5 A_i\right) = \sum_{i=1}^5 P(A_i) - \sum_{i < j} P(A_i \cap A_j) + \sum_{i < j < k} P(A_i \cap A_j \cap A_k) - \sum_{i < j < k < l} P(A_i \cap A_j \cap A_k \cap A_l) + P\left(\bigcap_{i=1}^5 A_i\right)$$

$$= 5P(A_1) - C_{5,2} P(A_1 \cap A_2) + C_{5,3} P(A_1 \cap A_2 \cap A_3) - C_{5,4} P(A_1 \cap A_2 \cap A_3 \cap A_4) + C_{5,5} P\left(\bigcap_{i=1}^5 A_i\right)$$

by symmetry: all $P(A_i)$ are same
 all $P(A_i \cap A_j)$ are same for $i \neq j$
 etc

$$= 5 \left(\frac{4}{5}\right)^{10} - 10 \left(\frac{3}{5}\right)^{10} + 10 \left(\frac{2}{5}\right)^{10} - 5 \left(\frac{1}{5}\right)^{10} + 0 = 0.4775$$

3.5 #2 Roll 2 dice. $A = \text{"at least one 6"}$, $B = \text{"sum is 9"}$. $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{2/36}{11/36} = \frac{2}{11}$

(Sum)	0	1	2	3	4	5	6
1		2	3	4	5	6	7
2		3	4	5	6	7	8
3		4	5	6	7	8	9
4		5	6	7	8	9	10
5		6	7	8	9	10	11
6		7	8	9	10	11	12

3.5 #4: Done in class

3.5 #7: $P(A) = 0.6$, $P(B) = 0.4$, $P(A \cap B) = 0.3$

$$P(A|A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A) + P(B) - P(A \cap B)} = \frac{6}{6 + 4 - 3} = \frac{6}{7}$$