Solutions HW2 (sec 1.7)

#32 $P(\text{Al wins 1 set}) = 0.7$, $P(\text{Bob wins 1 set}) = 0.3$

(a) $P(\text{Al wins first 2 sets}) = P(AA) + P(BAA) = (0.7)(0.7) + (0.3)\cdot(0.7)(0.7) = 0.637$

(b) $P(\text{Al wins first 3 sets}) = P(AAA) + \left(P(ABAA) + P(AABA) + P(BAAA)\right)$
$$+ \left(P(AABBA) + P(ABABA) + P(ABBAA) + P(BBAAA) + P(BABAA) + P(BAABA)\right)$$
$$= (0.7)^3 + 3(0.7)^3\cdot(0.3) + 6(0.7)^3(0.3)^2 = 0.837$$

#38 n children $\to P(\text{all } n \text{ children have same sex}) = 2\left(\frac{1}{2}\right)^n = \frac{1}{2^{n-1}}$

$\to P(\text{at least one of each sex}) = 1 - \frac{1}{2^{n-1}} \geq 0.95 \to n \geq \frac{\ln 20}{\ln 2} + 1 \simeq 5.3$

So $n \geq 6$.

#47 $P(\text{exactly 1 of 5 H and 4 of 5 T}) = \frac{10}{2^5} = \frac{5}{16} = p$

Number of tosses X until 1st success is geometric RV with $p = \frac{5}{16}$

$\to E(X) = \frac{1}{p} = \frac{16}{5} = 3.2$

#51 X = number of games that end World Series

$P(X=4) = 2\cdot\left(\frac{1}{2}\right)^4 = \frac{1}{8}$

$P(X=5) = 2\cdot\left(4\cdot\left(\frac{1}{2}\right)^5\right) = \frac{1}{4}$

$P(X=6) = 2\cdot\left(10\cdot\left(\frac{1}{2}\right)^6\right) = \frac{5}{16}$

$P(X=7) = 1 - \left(\frac{1}{8} + \frac{1}{4} + \frac{5}{16}\right) = \frac{5}{16}$

$E(X) = 4\cdot\frac{1}{8} + 5\cdot\frac{1}{4} + 6\cdot\frac{5}{16} + 7\cdot\frac{5}{16} = \frac{93}{16} = 5.8125$

$E(X^2) = 4^2\cdot\frac{1}{8} + 5^2\cdot\frac{1}{4} + 6^2\cdot\frac{5}{16} + 7^2\cdot\frac{5}{16} = \frac{557}{16}$

$\mathrm{Var}(X) = E(X^2) - (E(X))^2 = \frac{557}{16} - \frac{93^2}{16^2} \simeq 1.03$

#55 No, because $\mathrm{Var}(X) = E\left((X - E(X))^2\right)$ is always ≥ 0.

Here, $\mathrm{Var}(X) = E(X^2) - (E(X))^2 = 8 - 3^2 = -1 < 0$.