Solutions HW 10 Sec 6.7

19. $P(|\bar{X}_{10^4}-\frac{1}{2}| \geq 0.01) \leq \frac{Var(\bar{X}_{10^4})}{(0.01)^2} = \frac{1}{4}$

$P(|\bar{X}_{10^4}-\frac{1}{2}| \geq 0.01) = P(|\frac{S_n}{n}-\frac{1}{2}| \geq 0.01)$ where $S_n = X_1+\dots+X_n$, $n = 10^4$

$= P(\frac{1}{\sqrt{n}}|\frac{S_n - n\cdot\frac{1}{2}}{\sqrt{n}}| \geq 0.01) = P(\overbrace{|\frac{S_n - n\frac{1}{2}}{\sigma\sqrt{n}}|}^{\chi} \geq \frac{\sqrt{n}(0.01)}{\sigma})$ where $\sigma = \frac{1}{2}$

$\overset{CLT}{\simeq} P(|\chi| \geq \frac{\sqrt{10^4}(0.01)}{\frac{1}{2}}) = P(|\chi| \geq 2) = P(\chi \leq -2) + P(\chi \geq 2) =$

$2P(\chi \geq 2) = 2(1-P(\chi < 2)) = 2(1-\Phi(2)) = 0.0456$

20. $X \sim Pois(16)$ (a) $P(X \geq 28) = P(X-16 \geq 12) \leq \frac{Var(X)}{12^2} = \frac{16}{12^2} = 0.111$

(b) $P(X \geq 28) = P(\frac{X-16}{4} \geq 3) \overset{CLT}{\simeq} P(\chi \geq 3) = 0.0014$

21. $n = 100$, $S_n = \sum_{i=1}^{n} X_i$, X_i Bernoulli with $\mu = \frac{1}{2}$, $\sigma = \frac{1}{2}$

$P(|S_n - n\mu| \leq 3) = P(\overbrace{\frac{|S_n - n\mu|}{\sigma\sqrt{n}}}^{\chi} \leq \frac{3}{\frac{1}{2}\cdot 10}) \underset{CLT}{=} 0.4514$

24. $n = 162$, $S_n = \sum_{i=1}^{n} X_i$, X_i Bernoulli with $\mu = \sigma = \frac{1}{2}$

$P(S_n \geq 87) = P(\overbrace{\frac{S_n - n\mu}{\sigma\sqrt{n}}}^{\chi} \geq \frac{87 - 162\cdot\frac{1}{2}}{\frac{1}{2}\cdot\sqrt{162}}) \overset{CLT}{\simeq} 1 - \Phi(\frac{12}{\sqrt{162}})$

27. m such that $P(1250-m \leq S_n \leq 1250+m) = P(\frac{-m}{50\cdot\frac{1}{2}} \leq \frac{S_n - n\mu}{\sigma\sqrt{n}} \leq \frac{m}{50\cdot\frac{1}{2}})$

$\overset{CLT}{\simeq} \Phi(\frac{m}{25}) - \Phi(\frac{-m}{25}) = 2\Phi(\frac{m}{25}) - 1 = \frac{2}{3} \Rightarrow \Phi(\frac{m}{25}) = \frac{5}{6} \simeq 0.8340$

$\Rightarrow \frac{m}{25} = 0.97 \Rightarrow m = 24.25$

invert Φ (table p237) So, $m = 24$.