HW1 Section 1.7

10. $P(\text{sum}=9) = \dfrac{6+6+3+3+6+1}{6^3} = \dfrac{25}{6^3}$ So $P(\text{sum}=9) < P(\text{sum}=10)$

$P(\text{Sum}=10) = \dfrac{6+6+3+6+3+3}{6^3} = \dfrac{27}{6^3}$

16. $P(\{c\}) = P(\{a,b,c\}) - P(\{a,b\}) = 1 - 0.7 = 0.3$

$P(\{a\}) = P(\{a,b,c\}) - P(\{b,c\}) = 1 - 0.6 = 0.4$

$\Rightarrow P(\{b\}) = P(\{a,b,c\}) - P(\{a\}) - P(\{c\}) = 1 - 0.4 - 0.3 = 0.3$

20. Family with 3 kids. $A = \text{"at most 1 G"} = \{(B,B,B),(G,B,B),(B,G,B),(B,B,G)\}$

$B = \text{"both sexes"} = \{(B,B,B),(G,G,G)\}^C$

(a) $P(A) = \dfrac{|A|}{2^3} = \dfrac{4}{2^3} = \dfrac{1}{2}$, $P(B^c) = \dfrac{|B^c|}{2^3} = \dfrac{2}{2^3} = \dfrac{1}{4} \Rightarrow P(B) = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

$P(A \cap B) = P(\{(G,B,B),(B,G,B),(B,B,G)\}) = \dfrac{3}{2^3} = P(A) \cdot P(B) = \dfrac{1}{2} \cdot \dfrac{3}{4}$

So yes, A and B are independent

(b) 4 kids $\rightarrow$ $P(A) = \dfrac{4}{2^4} = \dfrac{1}{4}$, $P(B^c) = \dfrac{2}{2^4} = \dfrac{1}{8} \Rightarrow P(B) = 1 - \dfrac{1}{8} = \dfrac{7}{8}$

$P(A \cap B) = \dfrac{4}{2^4} = \dfrac{1}{4} \neq P(A)P(B) = \dfrac{1}{4} \cdot \dfrac{7}{8}$, so A, B <u>not</u> independent.

26. A, B independent. $P(A) = 0.4$, $P(A \cup B) = 0.64$

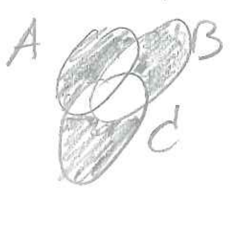
$\Rightarrow P(A \cup B) = \underline{0.64} = P(A) + P(B) - P(A \cap B) = 0.4 + P(B) - P(A)P(B)$

$= \underline{0.4 + (1-0.4)P(B)}$

So $0.64 = 0.4 + 0.6\,P(B) \Rightarrow P(B) = \dfrac{0.24}{0.6} = 0.4$

28. $P(A) = 1/4$, $P(B) = 1/3$, $P(C) = 1/2$ and A, B, C independent

$P(\text{exactly 1 of A,B,C occurs}) = P(A \cup B \cup C) - (P(A \cap B) + P(A \cap C) + P(B \cap C)) + 2P(A \cap B \cap C) = P(A) + P(B) + P(C) - 2(P(A \cap B) + P(B \cap C) + P(A \cap C)) + 3P(A)P(B)P(C)$



$= \dfrac{1}{4} + \dfrac{1}{3} + \dfrac{1}{2} - 2\left(\dfrac{1}{4} \cdot \dfrac{1}{3} + \dfrac{1}{4} \cdot \dfrac{1}{2} + \dfrac{1}{3} \cdot \dfrac{1}{2}\right) + 3 \cdot \dfrac{1}{4} \cdot \dfrac{1}{3} \cdot \dfrac{1}{2}$

$= \dfrac{3+4+6-(2+3+4)}{12} + \dfrac{1}{8} = \dfrac{1}{3} + \dfrac{1}{8} = \dfrac{11}{24}$