

- Split 2's, 3's, 6's, and 7's when the dealer's upcard is 3 through 7.
- Split 9's when the dealer's upcard is 2 through 9, but not 7.

The reason for splitting aces should be obvious. It is such a good play that it has on occasion been forbidden. Some casino rules do not allow further drawing after aces are split, and if a 10 lands on the ace, it is not a blackjack. To see why 8's are singled out for splitting, note that $8 + 8 = 16$, which wins only if the dealer busts, while an 8 paired with a 10 produces an 18.

Counting cards. Edward Thorp's book *Beat the Dealer*, which astonished the world in 1962 by demonstrating that by "counting cards" (that is, by keeping track of the difference between the numbers of cards you have seen that count 10 and those that count 2 through 6) and by adjusting your betting you can make money from blackjack. Before the reader plans a trip to Las Vegas or Atlantic City, we would like to point out that playing this strategy requires hardwork, that making money with it requires a lot of capital, and that casinos are allowed to ask you to leave if they think you are playing it. The book *Bringing Down the House* gives an entertaining account of MIT students using the strategy to win money at blackjack.

2.7 Exercises

Permutations and combinations

1. How many possible batting orders are there for nine baseball players?
2. A tire manufacturer wants to test four different types of tires on three different types of roads at five different speeds. How many tests are required?
3. 16 horses race in the Kentucky Derby. How many possible results are there for win, place, and show (first, second, and third)?
4. A school gives awards in five subjects to a class of 30 students but no one is allowed to win more than one award. How many outcomes are possible?
5. A tourist wants to visit six of America's ten largest cities. In how many ways can she do this if the order of her visits is (a) important or (b) not important?
6. Five businessmen meet at a convention. How many handshakes are exchanged if each shakes hands with all the others?
7. A commercial for Glade Plug-ins says that by inserting 2 of a choice of 11 scents into the device, you can make more than 50 combinations. If we exclude the boring choice of two of the same scent, how many possibilities are there?

8. In a class of 19 students, 7 will get A's. In how many ways can this set of students be chosen?
9. (a) How many license plates are possible if the first three places are occupied by letters and the last three by numbers? (b) Assuming all combinations are equally likely, what is the probability that the three letters and the three numbers are different?
10. How many four-letter "words" can you make if no letter is used twice and each word must contain at least one vowel (A, E, I, O, or U)?
11. Assuming all phone numbers are equally likely, what is the probability that all the numbers in a seven-digit phone number are different?
12. A domino is an ordered pair (m, n) with $0 \leq m \leq n \leq 6$. How many dominoes are in a set if there is only one of each?
13. A person has 12 friends and will invite 7 to a party. (a) How many choices are possible if Al and Bob are feuding and will not both go to the party? (b) How many choices are possible if Al and Betty insist that they both go or neither one goes?
14. A basketball team has 5 players more than 6 feet tall and 6 who are less than 6 feet. How many ways can they have their picture taken if the 5 taller players stand in a row behind the 6 shorter players who are sitting on a row of chairs?
15. The Duke basketball team has 10 women who can play guard and 12 tall women who can play the other three positions. At the start of the game, the coach gives the referee a starting lineup that lists who will play left guard, right guard, left forward, center, and right forward. In how many ways can this be done?
16. Six students, three boys and three girls, lineup in a random order for a photograph. What is the probability that the boys and girls alternate?
17. Seven people sit at a round table. How many ways can this be done if Mr. Jones and Miss Smith (a) must sit next to each other and (b) must not sit next to each other? (Two seating patterns that differ only by a rotation of the table are considered the same.)
18. How many ways can eight rooks be put on a chessboard so that no rook can capture any other rook? Or, what is the same: How many ways can 8 markers be placed on an 8×8 grid of squares so that there is at most one in each row or column?

19. A BINGO card is a 5×5 grid. The center square is a free space and has no number. The first column is filled with five distinct numbers from 1 to 15, the second with five numbers from 16 to 30, the middle column with four numbers from 31 to 45, the fourth with five numbers from 46 to 60, and the fifth with five numbers from 61 to 75. Since the object of the game is to get five in a row horizontally, vertically, or diagonally, the order is important. How many BINGO cards are there?

20. Continuing with the setup from the previous problem, in BINGO numbers are drawn from 1 to 75 without replacement. When a number is called you put a marker on that square. If you have five in a row horizontally, vertically, or diagonally, you have a BINGO. What is the probability you will have a BINGO after (a) four numbers are called? (b) After five?

Multinomial counting problems

21. How many different ways can the letters in the following words be arranged: (a) money, (b) banana, (c) statistics, (d) Mississippi?

22. 12 different toys are to be divided among 3 children so that each one gets 4 toys. How many ways can this be done?

23. A club with 50 members is going to form two committees, one with 8 members and the other with 7. How many ways can this be done (a) if the committees must be disjoint? (b) If they can overlap?

24. If seven dice are rolled, what is the probability that each of the six numbers will appear at least once?

25. How many ways can 5 history books, 3 math books, and 4 novels be arranged on a shelf if the books of each type must be together?

26. Suppose three runners from team A and three runners from team B have a race. If all six runners have equal ability, what is the probability that the three runners from team A will finish first, second, and fourth?

27. Four men and four women are shipwrecked on a tropical island. How many ways can they (a) form four male–female couples, (b) get married if we keep track of the order in which the weddings occur, (c) divide themselves into four unnumbered pairs, (d) split up into four groups of two to search the North, East, West, and South shores of the island, (e) walk single file up the ramp to the ship when they are rescued, (f) take a picture to remember their ordeal if all eight stand in a line but each man stands next to his wife?

Binomial and multinomial distributions

28. A die is rolled 8 times. What is the probability that we will get exactly two 3's?
29. Mary knows the answers to 20 of the 25 multiple-choice questions on the Psychology 101 exam, but she has skipped several of the lectures; she must take random guesses for the other five. Assuming each question has four answers, what is the probability that she will get exactly 3 of the last 5 questions right?
30. In 1997, 10.8% of female smokers smoked cigars. In a sample of size 10 female smokers, what is the probability that (a) exactly 2 of the women smoke cigars? (b) At most 1 smokes cigars?
31. A 1994 report revealed that 32.6% of U.S. births were to unmarried women. A parenting magazine selected 30 women who gave birth in 1994 at random. (a) What is the probability that exactly 10 of the women were unmarried? (b) Using your calculator determine the probability that in the sample at most 10 are unmarried.
- ✕ 32. 20% of all students are left-handed. A class of size 20 meets in a room with 5 left-handed and 18 right-handed chairs. Use your calculator to find the probability that each student will have a chair to match their needs.
33. David claims to be able to distinguish brand B beer from brand H, but Alice claims that he just guesses. They set up a taste test with 10 small glasses of beer. David wins if he gets 8 or more right. What is the probability that he will win (a) if he is just guessing? (b) If he gets the right answer with probability 0.9?
- ✕ 34. The following situation comes up the game of Yahtzee. We have three rolls of five dice and want to get three sixes or more. On each turn we reroll any dice that are not 6's. What is the probability that we succeed?
35. A baseball pitcher throws a strike with probability 0.5 and a ball with probability 0.5. He is facing a batter who never swings at a pitch. What is the probability that he strikes out, that is, gets three strikes before four balls?
- ✕ 36. A baseball player is said to "hit for the cycle" if he has a single, a double, a triple, and a home run all in one game. Suppose these four types of hits have probabilities $1/6$, $1/20$, $1/120$, and $1/24$. What is the probability of hitting for the cycle if he gets to bat (a) four times and (b) five times? (c) Using $P(\cup_i A_i) \leq \sum_i P(A_i)$ shows that the answer to (b) is at most 5 times the answer to (a). What is the ratio of the two answers?

Poisson approximation

37. Compare the Poisson approximation with the exact binomial probabilities when (a) $n = 10$, $p = 0.1$, (b) $n = 20$, $p = 0.05$, and (c) $n = 40$, $p = 0.025$.
38. Use the Poisson approximation to compute the probability that you will roll at least one double 6 in 24 trials. How does this compare with the exact answer?
39. The probability of a three of a kind in poker is approximately $1/50$. Use the Poisson approximation to compute the probability that you will get at least one three of a kind if you play 20 hands of poker.
40. Calls to a toll-free hotline service are made randomly at rate 2 per minute. The service has five operators, none of whom is currently busy. Use the Poisson distribution to estimate the probability that in the next minute there are < 5 calls.
41. In one of the New York state lottery games, a number is chosen at random between 0 and 999. Suppose you play this game 250 times. Use the Poisson approximation to estimate the probability that you will never win and compare this with the exact answer.
42. If you bet \$1 on number 13 at roulette (or on any other number) then you win \$35 if that number comes up, an event of probability $1/38$, and you lose your dollar otherwise. Suppose you play 70 times. Use the Poisson approximation to estimate the probability that (a) you have won 0 times and lost \$70, and (b) you have won 1 time and lost \$34. (c) If you win 2 times you have won \$2. Combine the results of (a) and (b) to conclude that the probability that you will have won more money than you have lost is larger than $1/2$.
43. In a particular Powerball drawing 210,850,582 tickets were sold. The chance of winning the lottery is 1 in 80,000,000. Use the Poisson approximation to estimate the probability that there is exactly one winner.
44. Suppose that the probability of a defect in a foot of magnetic tape is 0.002. Use the Poisson approximation to compute the probability that a 1,500-foot roll will have no defects.
45. Suppose 1% of a certain brand of Christmas lights is defective. Use the Poisson approximation to compute the probability that in a box of 25 there will be at most one defective bulb.
46. In February 2000, 2.8% of Colorado's labor force was unemployed. Calculate the probability that in a group of 50 workers exactly one is unemployed.

47. An insurance company insures 3,000 people, each of whom has a $1/1,000$ chance of an accident in 1 year. Use the Poisson approximation to compute the probability that there will be at most 2 accidents.
48. Suppose that 1% of people in the population are more than 6 feet 3 inches tall. What is the chance that in a group of 200 people picked at random from the population at least four people will be more than 6 feet 3 inches tall.
49. In an average year in Mythica there are 8 fires. Last year there were 12 fires. How likely is it to have 12 or more fires just by chance?
- X 50. An airline company sells 160 tickets for a plane with 150 seats, knowing that the probability that a passenger will not show up for the flight is 0.1. Use the Poisson approximation to compute the probability that they will have enough seats for all the passengers who show up.
- X 51. Books from a certain publisher contain an average of 1 misprint per page. What is the probability that on at least one page in a 300-page book there are five misprints?

Urn problems

52. Two red cards and two black cards are lying face down on the table. You pick two cards and turn them over. What is the probability that the two cards are different colors?
53. Four people are chosen at random from 5 couples. What is the probability that two men and two women are selected?
54. You pick 5 cards out of a deck of 52. What is the probability that you get exactly 2 spades?
55. Seven students are chosen at random from a class with 17 boys and 13 girls. What is the probability that 4 boys and 3 girls are selected?
56. In a carton of 12 eggs, 2 are rotten. If we pick 4 eggs to make an omelet, what is the probability that we do not get a rotten egg?
57. An electronics store receives a shipment of 30 calculators of which 4 are defective. Six of these calculators are selected to be sent to a local high school. What is the probability that exactly one is defective?
58. A scrabble set contains 54 consonants, 44 vowels, and 2 blank tiles. Find the probability that your initial draw contains 5 consonants and 2 vowels.
- * 59. (a) How many ways can we pick 4 students from a group of 30 to be on the math team? (b) Suppose there are 18 boys and 12 girls. What is the probability that the team will have 2 boys and 2 girls.

60. The following probability problem arose in a court case concerning possible discrimination against black nurses. 26 white nurses and 9 black nurses took an exam. All the white nurses passed but only 4 of the black nurses did. What is the probability that we would get this outcome if the five nurses who failed were chosen at random?
61. A closet contains 8 pairs of shoes. You pick out 5. What is the probability of (a) no pair, (b) exactly one pair, and (c) two pairs?
62. A drawer contains 10 black, 8 brown, and 6 blue socks. If we pick two socks at random, what is the probability that they match?
63. A dance class consists of 12 men and 10 women. Five men and five women are chosen and paired up to dance. In how many ways can this be done?
64. Suppose we pick 5 cards out of a deck of 52. What is the probability that we get at least one card of each suit?
65. A bridge hand in which there is no card higher than a 9 is called a *Yarborough* after the Earl who liked to bet at 1,000 to 1 that your bridge hand would have a card that was 10 or higher. What is the probability of a Yarborough when you draw 13 cards out of a deck of 52.
66. Two cards are a blackjack if one is an A and the other is a K, Q, J, or 10. (a) If you pick two cards out of a deck, what is the probability that you will get a blackjack? (b) Suppose you are playing blackjack against the dealer with a freshly shuffled deck. What is the probability that you or the dealer will get a blackjack?
67. A student studies 12 problems from which the professor will randomly choose 6 for a test. If the student can solve 9 of the problems, what is the probability she can solve at least 5 of the problems on the test?
68. A football team has 16 seniors, 12 juniors, 8 sophomores, and 4 freshmen. (a) If we pick 5 players at random, what is the probability that we will get 2 seniors and 1 from each of the other 3 classes? (b) What is the most likely outcome?
69. In a kindergarten class of 20 students, one child is picked each day to help serve the morning snack. What is the probability that in 1 week five different children are chosen?
70. An investor picks 3 stocks out of 10 recommended by his broker. Of these, 6 will show a profit in the next year. What is the probability that the investor will pick (a) 3, (b) 2, (c) 1, (d) 0 profitable stocks?
71. Four red cards (that is, hearts and diamonds) and four black cards are face down on the table. A psychic who claims to be able to locate the four black cards turns over 4 cards and gets 3 black cards and 1 red card. What is the probability that he would get 3 or 4 black cards if he were guessing?

72. A town council considers the question of closing down an "adult" theater. The five men on the council all vote against this and the three women vote in favor. What is the probability that we would get this result (a) if the council members determined their votes by flipping a coin? (b) If we assigned the five "no" votes to council members chosen at random?

73. An urn contains white balls numbered 1 to 15 and black balls also numbered 1 to 15. Suppose you draw 4 balls. What is the probability that (a) no two have the same number? (b) You get exactly one pair with the same number? (c) You get two pair with the same numbers?

74. A town has four TV repairmen. In the first week of September four TV sets break and their owners call repairmen chosen at random. Find the probability that the number of repairmen who have jobs is 1, 2, 3, 4.

75. Compute the probabilities of the following poker hands when we roll five six-sided dice.

(a) Five of a kind	0.000771
(b) Four of a kind	0.019290
(c) A full house	0.038580
(d) Three of a kind	0.154320
(e) Two pair	0.231481
(f) One pair	0.462962
(g) No pair	0.092592

76. In seven-card stud you receive seven cards and use them to make the best poker hand you can. Ignoring the possibility of a straight or a flush the probability that the best hand you can make with your cards is

	Seven cards	Five cards
(a) Four of a kind	0.001680	0.000240
(b) A full house	0.025968	0.001441
(c) Three of a kind	0.049254	0.021128
(d) Two pair	0.240113	0.047539
(e) One pair	0.472839	0.422569
(f) No pair	0.210150	0.507082

Verify the probabilities for seven-card stud. Hint: For full house you need to consider hand patterns: 3-3-1 and 3-2-2 in addition to the more likely 3-2-1-1. For two pair you also have to consider the possibility of three pair.

Probabilities of unions

77. Six high school teams play each other in the Southern Tier division. Each team plays all the other teams once. What is the probability that some team has a perfect 5–0 season?
78. Suppose you draw 7 cards out of a deck of 52. What is the probability that you will have (a) exactly 5 cards of one suit? (b) At least 5 cards of one suit?
79. In a certain city 60% of the people subscribe to newspaper A, 50% to B, 40% to C, 30% to A and B, 20% to B and C, and 10% to A and C, but no one subscribes to all three. What percentage subscribe to (a) at least one newspaper, (b) exactly one newspaper, (c) only to A, (d) only to B?
80. Santa Claus has 45 drums, 50 cars, and 55 baseball bats in his sled. 15 boys will get a drum and a car, 20 a drum and a bat, 25 a bat and a car, and 5 will get three presents. (a) How many boys will receive presents? (b) How many boys will get just a drum?
81. Use the inclusion–exclusion formula to compute the probability that a randomly chosen number between 0000 and 9999 contains at least one 1. Check this by computing the probability that there is no 1.
82. Ten people call an electrician and ask him to come to their houses on randomly chosen days of the work week (Monday through Friday). What is the probability of $A =$ “he has at least one day with no jobs”?
83. We pick a number between 0 and 999, then a computer picks one at random from that range. Use (2.15) to compute the probability that at least two of our digits will match the computer’s number. (Note: We include any leading zeros, so 017 and 057 have two matching digits.)
84. You pick 13 cards out of a deck of 52. What is the probability that you will not get a card from every suit?
85. You pick 13 cards out of a deck of 52. Let $A =$ “you have exactly 6 cards in at least one suit” and $B =$ “you have exactly 6 spades.” The first Bonferroni inequality says that $P(A) \leq 4P(B)$. Compute $P(A)$ and $P(A)/P(B)$.
86. Use the first two Bonferroni inequalities to compute an upper and a lower bound on the probability that in a group of 60 people, at least 3 were born on the same day.

87. Suppose we roll two dice 6 times. Use the first three Bonferroni inequalities to compute bounds on the probability of $A =$ "we get at least one double 6." Compare the bounds with the exact answer $1 - (35/36)^6$.
88. Suppose we try 20 times for an event with probability 0.01. Use the first three Bonferroni inequalities to compute bounds on the probability of one success.