Final Exam MTH 361*

December 8, 2016

Name:

This is a **closed book** exam and the use calculators is **not** permitted. You can use a **1-sided formula sheet with 25 formulas**. I will check your sheets during the final.

The first 5 problems are multiple choice problems, each worth 5 points. Please circle the right answer (there is exactly one for each problem). The next 5 problems are worth 10 points each, and will be graded with partial credit. Make sure to accurately justify your answers for these problems.

1. Suppose that X and Y have the following distribution

X	Y= -1	0	1
1	0	0.25	0
0	0.25	0.25	0.25

   Then

   - The covariance of X and Y is nonzero, and X and Y are independent.
   - The covariance of X and Y is zero, and X and Y are independent.
   - The covariance of X and Y is zero, and X and Y are not independent.
   - The covariance of X and Y is nonzero, and X and Y are not independent.

2. Pick 2 cards from a deck of 52 cards. The probability that the second card is a King is

   - 4/51
   - 1/13
   - 3/52
   - None of the above.

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3. Suppose that X is a Poisson random variable with parameter $\lambda = 4$. Use Chebyshev's inequality to estimate $P(|X - 4| \geq 2)$, and also determine the exact probability.

   - Estimate 1; Exact $1 - \sum_{k=3}^{5} e^{-4} \frac{4^k}{k!}$
   - Estimate 1; Exact $1 - \sum_{k=2}^{6} e^{-4} \frac{4^k}{k!}$
   - Estimate 1/2; Exact $1 - \sum_{k=3}^{5} e^{-4} \frac{4^k}{k!}$
   - Estimate 1/2; Exact $1 - \sum_{k=2}^{6} e^{-4} \frac{4^k}{k!}$

4. Let k be a given positive integer. Let T_k be the number of independent trials that are needed to obtain k successes, where the probability that one trial is successful is p. Then $E(T_k)$ equals:

   - $k(1 - p)$
   - kp
   - k/p
   - $k/(1 - p)$

5. In the College of Science of OSU, 60% of the scientists read Nature, 50% read Science and 40% read PNAS, 30% read Nature and Science, 20% read Science and PNAS, and 10% read Nature and PNAS. No scientists read all 3 journals. What percentage of scientists read at least one of these journals?

   - 70%
   - 80%
   - 85%
   - 90%

6. In a course with a MWF schedule, a student goes to class on M and on W with probability 0.8, but only with probability 0.4 on F. What is the probability that today is F, given that the student is in class?

7. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables with distribution function $F(x)$.

   (a) Find the distribution function of the random variable $Y = \min\{X_1, X_2, \ldots, X_n\}$.

   (b) Suppose that $X_1, X_2, \ldots, X_n$ are exponentially distributed random variables with parameter λ. Then what is the distribution function of the random variable $Y = \min\{X_1, X_2, \ldots, X_n\}$?

8. The Golden State Warriors and the Cleveland Cavaliers played the NBA Finals. (This is a best of 7 series; the team that wins 4 games first, wins the finals). Assuming that the Warriors win a game with probability 0.45, and the Cavaliers win a game with probability 0.55, what is the probability that the Cavaliers win the Finals?

9. In the 2015-2016 NBA season the Golden State Warriors set a record by winning 73 games, and losing only 9 games. Assuming that they win each game with probability 0.9, and that the result of all 82 games played is independent,

   - Give an exact formula for the probability of winning at least 73 games out of 82 games played.
   - Approximate the probability above by using a normal approximation. (Write your answer in terms of Φ, the distribution function of the standard normal distribution, but do not attempt to evaluate this function.)[1]

10. Suppose that X and Y have joint density function $f(x, y) = 2$ for $0 < x < y < 1$. Calculate $P(Y - X > z)$, where z is an arbitrary real number.

[1]Recall that $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2}\, dy$