Solutions toExam 2: MTG 3214*

November 4, 2008

Name:
Student ID:

This is a **closed book** exam and the use of calculators is **not** allowed.

1. Prove: The feet of the perpendiculars from a point to the sides of a triangle are collinear if and only if the point lies on the circumcircle.

   Solution: This is Theorem 2.51.

2. Prove that the nine point circle of the triangle formed by connecting the excenters of a given triangle ABC, is the circumcircle of triangle ABC.

   Solution: (This is problem 1.8.3 which was a monthly HW problem)

   Consider Fig 1.4B on p.12. The 9 point circle of $\triangle I_a I_b I_c$ must certainly contain the 3 feet of its altitudes. I claim that these feet are A, B and C. To prove this, notice that the internal and external bisector of any angle are always perpendicular. (convince yourself by drawing a picture if you don't see this!).

   On the other hand, there is only 1 circle that contains the 3 points A, B and C, namely the circumcircle of $\triangle ABC$. Consequently, this circumcircle must be the 9 point circle of $\triangle I_a I_b I_c$ because we know that the latter contains A, B and C.

3.
   - Define the power of a point with respect to a circle. Using this definition, give 3 examples (draw figures) that show that the power of a point with respect to a circle can be negative, zero or positive.

     $$\text{Power of P with respect to circle} = d^2 - R^2,$$

     where d is the distance between the center of the circle and P, and R is the radius of the circle. Figure 2.1A and 2.1B are examples of a point having negative and positive power respectively (for a zero power draw P on the circle).
   - What is a Simson line? A degenerate pedal triangle, associated to a given triangle. This happens if (and only if) the pedal point belongs to the circumcircle of the given triangle. This is exactly what I asked you to prove in the first problem of this exam.
   - Let three distinct circles C_1, C_2 and C_3 be given. True of false: The radical center of the three circles always exists (prove if true; give counterexample if false).

     False, there are several possible counterexamples. Here's one: Pick 3 circles having pairwise distinct centers that belong to a line (the centers are collinear). For each pair of circles the radical axis is perpendicular to this line, so the 3 radical axes will be parallel. This implies they don't intersect, and thus the radical center does not exist.
   - Let the power of a point with respect to some circle be positive. Interpret this power geometrically. It is the square of the length of the tangent.
   - The orthic triangle of a given triangle ABC is the pedal triangle with respect to a certain point. Which point? Explain your answer.

     It is the orthocenter of the triangle ABC.

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