# Practice Exam 4: MAP 4305*

1. It is known that Hermite's equation

$$y'' - 2xy' + 2ny = 0,$$

where $n$ is a nonnegative integer has polynomial solutions of degree $n$. Denote these solutions by $H_n(x)$. There is a generating function for these polynomials:

$$e^{2tx-t^2} = \sum_{n=0}^{\infty} H_n(x)\frac{t^n}{n!}$$

- Using the generating function, determine $H_0(x)$, $H_1(x)$ and $H_2(x)$.
- Let $n = 2$ and find $H_2(x)$ by solving Hermite's equation directly (notice that $x = 0$ is an ordinary point).

2. Find (graphically and approximately) the **positive** eigenvalues and eigenfunctions of the following eigenvalue problem:

$$y'' + \lambda y = 0, \;\; y(0) = y(\pi) - y'(\pi) = 0.$$

Provide an open interval of the form $(0, c)$ for some to be determined $c > 0$ which contains $\lambda_1$, the smallest positive eigenvalue.

3. Find the adjoint problem of

$$x^2y'' - xy' + 3y = 0, \;\; y(1) = y(2) = 0.$$

Is it self-adjoint?

4. Consider the linear operator

$$L[y] = y^{(4)},$$

defined on the set of functions having continuous derivatives up to order 4, that satisfy the following periodic boundary conditions:

$$y^{(i)}(0) = y^{(i)}(1), \;\; i = 0, 1, 2, 3.$$

Show that $L$ is self-adjoint, ie show that

$$(L[y_1], y_2) = (y_1, L[y_2])$$

for all $y_1$ and $y_2$ in the domain of $L$ where $(y_1, y_2)$ denotes the usual inner product of the functions $y_1$ and $y_2$.

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