Practice Exam 2: MAP 4305*

1. Determine stability of the equilibrium at $(0,0)$ of

$$\begin{aligned} \dot{x} &= y^3 - 2x^3 \\ \dot{y} &= -3x - y^3 \end{aligned}$$

2. Let

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}, b = \begin{pmatrix} 1 \\ t \end{pmatrix}, x_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

First find e^{tA}. Next, using the variation of constants formula, solve

$$\dot{x} = Ax + b, \quad x(0) = x_0$$

3. Let $V : \mathbb{R}^2 \to \mathbb{R}$ be a twice continuously differentiable function. Consider the system

$$\begin{aligned} \dot{x} &= V_x(x,y) \\ \dot{y} &= V_y(x,y) \end{aligned}$$

Let (x^*, y^*) be a critical point of V (ie $V_x(x^*, y^*) = V_y(x^*, y^*) = 0$). Then (x^*, y^*) is clearly an equilibrium point of the system. Can it be a spiral (stable or unstable) or a center?

4. Find the nullclines and equilibria of

$$\begin{aligned} \dot{x} &= x(1 - x - 0.5y) \\ \dot{y} &= y(1 - 0.5x - y) \end{aligned}$$

Linearize at each equilibrium to determine its nature, and perform phase plane analysis.

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