Answers practice exam I: MAP 4305*

September 16, 2007.

Name:
Student ID:

This is a **closed book** exam and the use of calculators is **not** allowed.

1. $$A^{231} = TD^{231}T^{-1}, \;\; T = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \;\; D^{231} = \begin{pmatrix} (-1)^{231} & 0 \\ 0 & 3^{231} \end{pmatrix}, \;\; T^{-1} = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

   System is unstable.

2. $$P = \begin{pmatrix} 0.5 & 0 \\ 0.5 & 1 \end{pmatrix}, \;\; \text{Pagerank} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

3. Since $m(t) = x_1(t)y_2(t) - x_2(t)y_1(t)$, it follows that

   $$\dot{m}(t) = \dot{x}_1(t)y_2(t) + x_1(t)\dot{y}_2(t) - \dot{x}_2(t)y_1(t) - x_2(t)\dot{y}_1(t).$$

   Using the fact that

   $$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \text{ and } \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$$

   are solutions of the system, we can simplify this to:

   $$\dot{m}(t) = (a+d)(x_1(t)y_2(t) - x_2(t)y_1(t)) = (a+d)m(t).$$

   Solve this first order differential equation like you were taught in MAP2302, and you get

   $$m(t) = \mathrm{e}^{(a+d)t}\, m(0).$$

4. $$x(t) = X(t)c, \;\; X(t) = \begin{pmatrix} \mathrm{e}^{-t} & \mathrm{e}^{2t} \\ 0 & 3\,\mathrm{e}^{2t} \end{pmatrix}, \;\; c = \begin{pmatrix} \mathrm{e} - \frac{2}{3}\,\mathrm{e}^{-3} \\ \frac{2}{3}\,\mathrm{e}^{-2} \end{pmatrix}$$

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