1. exactness of $\underbrace{(x+y\,e^{2xy})}_{M}dx+\underbrace{nx\,e^{2xy}}_{N}dy=0$

$M_y = N_x$? $e^{2xy} + 2xye^{2xy} = ne^{2xy} + n\cdot 2xye^{2xy} \Leftrightarrow \boxed{n=1}$

$F_x = M = x + ye^{2xy} \Rightarrow F = \frac{x^2}{2} + \frac{1}{2}e^{2xy} + C(y)$

$F_y = N \Rightarrow xe^{2xy} + C'(y) = xe^{2xy} \Rightarrow C'(y) = 0$ or $C(y) = \underset{\text{real constant}}{C}$ $\Big\} \Rightarrow \boxed{F(x,y) = \frac{x^2}{2} + \frac{1}{2}e^{2xy} + C}$

2. Bernoulli: $\frac{dy}{dx} + p(x)y = q(x)y^n$

$z = y^{1-n} \Rightarrow \frac{dz}{dx} = (1-n)y^{-n}\frac{dy}{dx} = \frac{(1-n)}{y^n}\cdot\left[-p(x)y + q(x)y^n\right] = -(1-n)p(x)\overset{=z}{\boxed{y^{1-n}}} + (1-n)q(x)$

$\Rightarrow \frac{dz}{dx} = -(1-n)p(x)z + (1-n)q(x)$, linear equation. (*)

Solve: $y' + xy = xy^4$, so $n=4$, $p(x) = q(x) = x$

$z = y^{-3} \overset{(*)}{\Rightarrow} \frac{dz}{dx} = 3xz + (-3)x$ or $\frac{dz}{dx} - (3x)z = -3x$, linear equation

integrating factor: $\mu(x) = e^{\int -3x\,dx} = e^{-\frac{3x^2}{2}} \Rightarrow \mu(x)\frac{dz}{dx} - \mu(x)\cdot 3x\cdot z = -3x\mu(x)$

$$(\mu(x)z)' = -3xe^{-\frac{3x^2}{2}}$$

$\Rightarrow \mu(x)z = \int -3xe^{-\frac{3x^2}{2}}dx = \int e^{-\frac{3x^2}{2}}d\left(-\frac{3x^2}{2}\right) = e^{-\frac{3x^2}{2}} + C \Rightarrow z(x) = \frac{1}{\mu(x)}\left(e^{-\frac{3x^2}{2}} + C\right) = 1 + Ce^{\frac{3x^2}{2}}$

$\Rightarrow \boxed{y(x) = (z(x))^{-1/3} = \frac{1}{\left(1 + Ce^{\frac{3x^2}{2}}\right)^{1/3}}}$