Practice Exam 1: MAP 2302*

1. Let x be the independent, and y the dependent variable. Classify the following differential equations as separable, linear, exact, homogeneous, or none. Some equations may fit in several classes, and you should list all possibilities. **Do not try to solve these equations.**

   (a) $x^4 dx + y dy = 0$

   (b) $x dx + y^4 dy = 0$

   (c) $y^4 dx + x dy = 0$

   (d) $y dx + x^4 dy = 0$

   (e) $(xy - y^2)dx + x^2 dy = 0$

2. Solve the following equation:

$$(y\ln(x) + \mathrm{e}^x\, y^2)dx + (x\ln(x) - x + 2\,\mathrm{e}^x\, y + \sin(y))\, dy = 0$$

3. Does the following relation determine an implicit solution of the ODE?

$$x^2 + \ln(\frac{x}{y}) = 1, \quad \frac{dy}{dx} = \frac{y + 2x^2 y}{x}.$$

4. Assume that a salty solution runs through a tank with inflow rate 2 l/s, and outflow rate 1 l/s. The volume of the tank is 1000 l, and initially there is no salt, and 100 l of pure water in the tank. The input concentration of salt is 1 kg/l. Determine the amount (in kg) of salt in the tank at the time when the tank is completely filled. Recall that the ODE describing the amount of salt in the tank is:

$$\frac{dx}{dt} = F_1 c_{in}(t) - F_2 \frac{x}{V(t)},$$

   where F_1 (l/s) and F_2 (l/s) are inflow and outflow rates, $V(t)$ (in l) is the volume of the solution at time t, and $c_{in}(t)$ (in kg/l) is the concentration of salt in the solution at the input of the tank.

5. Consider the following population model:

$$\frac{dp}{dt} = p(1-p), \quad p(0) = p_0 > 0.$$

   (a) Solve this initial value problem.

   (b) Sketch the direction field and discuss what happens to solutions when $t \to \infty$. Does your answer depend on the value of p_0?

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