Preface Homework 2

Due 1/13/17 4 pm

NOTE: If you are gong to the CuWIP conference, you have an automatic extension until Tuesday at 4 pm for this assignment. Please write CuWIP at the top of your homework.

PRACTICE:

1. **QUIZ:** At the beginning of Friday's class, you will be expected to find the eigenvalues and eigenvectors of an arbitrary 2×2 matrix.

2. Find the eigenvectors and eigenvalues of all of the matrices from the Linear Transformations small group activity from Tuesday's class. Make up some 2×2 and 3×3 matrices of your own if you need more practice.

3. For practice with unit circle trigonometry, try Khan Academy at:

   khanacademy.org/math/trigonometry/unit-circle-trig-func/

   Also, try the unit circle simulation in the middle of the pages at:

   https://www.mathsisfun.com/sine-cosine-tangent.html

REQUIRED:

4. The orthogonal matrix
$$R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
   corresponds to a rotation around the z-axis by the angle θ.

   (a) Find the eigenvalues of this matrix.

   (b) Find the normalized eigenvectors of this matrix.

   (c) Describe how the eigenvectors do or do not correspond to the vectors which are held constant or "only stretched" by this transformation.

5. (a) By drawing pictures, convince yourself that the arbitrary unit vector $\hat{n}$ can be written as:
$$\hat{n} = \sin\theta\cos\phi\,\hat{\imath} + \sin\theta\sin\phi\,\hat{\jmath} + \cos\theta\,\hat{k}$$
   where θ and ϕ are the parameters used to describe spherical coordinates.

   (b) Find the entries of the matrix $\hat{n}\cdot\vec{\sigma}$ where the "matrix-valued-vector" $\vec{\sigma}$ is given in terms of the Pauli spin matrices by
$$\vec{\sigma} = \sigma_x\,\hat{\imath} + \sigma_y\,\hat{\jmath} + \sigma_z\,\hat{k}$$
   and $\hat{n}$ is given in part (a) above.

6. Consider the arbitrary Pauli matrix $\sigma_n = \hat{n} \cdot \vec{\sigma}$ where $\hat{n}$ is the unit vector pointing in an arbitrary direction.

   (a) Find the eigenvalues and normalized eigenvectors for σ_n.
   The answer is:

$$\begin{pmatrix} \cos\frac{\theta}{2}e^{-i\phi/2} \\ \sin\frac{\theta}{2}e^{i\phi/2} \end{pmatrix} \qquad\qquad \begin{pmatrix} -\sin\frac{\theta}{2}e^{-i\phi/2} \\ \cos\frac{\theta}{2}e^{i\phi/2} \end{pmatrix}$$

   It is not sufficient to show that this answer is correct by pluging into the eigenvalue equation. Rather, you should do all the steps of finding the eigenvalues and eigenvectors as if you don't know the answer. Hint: $\sin\theta = \sqrt{1-\cos^2\theta}$.

   (b) Show that the eigenvectors from part (a) above are orthogonal.

   (c) Simplify your results from part (a) above by considering the three separate special cases $\hat{n} = \hat{\imath}$, $\hat{n} = \hat{\jmath}$, $\hat{n} = \hat{k}$. In this way, find the eigenvectors and eigenvalues of σ_x, σ_y, and σ_z.

7. On the following diagrams, mark both θ and $\sin\theta$ for $\theta_1 = \frac{5\pi}{6}$ and $\theta_2 = \frac{7\pi}{6}$. Write one to three sentences about how these two representations are related to each other. (For example, see: www.mathsisfun.com/algebra/trig-interactive-unit-circle.html)