Preface Homework 1

Due 1/11/17 4 pm

PRACTICE:

1. **QUIZ:** There will be a brief quiz on Wednesday. You will be asked to multiply two 3×3 matrices, find the determinant of a 3×3 matrix, and find the square and the norm squared of a complex number in both rectangular and exponential forms.

2. For practice with unit circle trigonometry, try Khan Academy at:

   khanacademy.org/math/trigonometry/unit-circle-trig-func/

   Also, try the unit circle simulation in the middle of the pages at:

   https://www.mathsisfun.com/sine-cosine-tangent.html

3. Calculate the following quantities for the matrices:

$$A \doteq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \qquad B \doteq \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & j \end{pmatrix} \qquad C \doteq \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

   and the vectors:

$$|D\rangle \doteq \begin{pmatrix} 1 \\ i \\ -1 \end{pmatrix} \qquad |E\rangle \doteq \begin{pmatrix} 1 \\ i \end{pmatrix} \qquad |F\rangle \doteq \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

   (a) AB

   (b) $\mathrm{tr}(AB)$

   (c) $A^\dagger$

   (d) C^{-1}

   (e) $A|D\rangle$

   (f) $|E\rangle^\dagger \equiv \langle E|$

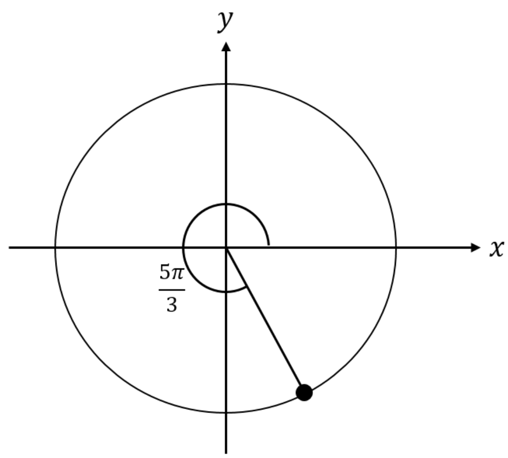
   (g) $\langle D|A|D\rangle$

   (h) $\det(\lambda\mathcal{I} - A)$ where λ is a scalar.

   (i) $(A|D\rangle)^\dagger$

   (j) Using explicit matrix multiplication (without using a theorem) verify that $(A|D\rangle)^\dagger = \langle D|A^\dagger$

REQUIRED:



4. Find the rectangular coordinates of the point where the angle $\frac{5\pi}{3}$ meets the unit circle. If this were a point in the complex plane, what would be the rectangular and exponential forms of the complex number? (See figure.)

5. Use Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$ and its complex conjugate to find formulas for $\sin\theta$ and $\cos\theta$. In your physics career, you will often need to read these formula "backwards," i.e. notice one of these combinations of exponentials in a sea of other symbols and say, "Ah ha! that is $\cos\theta$." So, pay attention to the result of the homework problem!

6. The Pauli spin matrices σ_x, σ_y, and σ_z are defined by:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

These matrices are related to angular momentum in quantum mechanics. Prove, and become familiar with, the identities listed below.

   (a) Show that each of the Pauli matrices is hermitian. (A matrix is hermitian if it is equal to its hermitian adjoint.

   (b) Show that the determinant of each of the Pauli matrices is -1.

   (c) Show that $\sigma_i^2 = \mathcal{I}$ for each of the Pauli matrices, i.e. for $i \in \{x, y, z\}$.

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These matrices are related to angular momentum in quantum mechanics. Prove, and become familiar with, the identities listed below.

(a) Show that $\sigma_x \sigma_y = i\sigma_z$ and $\sigma_y \sigma_x = -i\sigma_z$. (Note: These identities also hold under a cyclic permutation of $\{x, y, z\}$, e.g. $x \to y$, $y \to z$, and $z \to x$).

(b) The commutator of two matrices A and B is defined by $[A, B] \stackrel{\text{def}}{=} AB - BA$. Show that $[\sigma_x, \sigma_y] = 2i\sigma_z$. (Note: This identity also holds under a cyclic permutation of $\{x, y, z\}$, e.g. $x \to y$, $y \to z$, and $z \to x$).

(c) The anti-commutator of two matrices A and B is defined by $\{A, B\} \stackrel{\text{def}}{=} AB + BA$. Show that $\{\sigma_x, \sigma_y\} = 0$. (Note: This identity also holds under a cyclic permutation of $\{x, y, z\}$, e.g. $x \to y$, $y \to z$, and $z \to x$).

8. Consider the following matrices:

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(a) Explain what each of the matrices “does” geometrically when thought of as a linear transformation acting on a vector.

(b) The commutator of two matrices A and B is defined by $[A, B] \stackrel{\text{def}}{=} AB - BA$. Find the following commutators: $[A, B]$, $[A, C]$, $[B, C]$.