## Central Forces

## Quantum Calculations on a Ring I

Your group will be given one of the following normalized abstract quantum states on a ring:

$$
\begin{array}{rlrl}
\left|\Phi_{a}\right\rangle & =\frac{\sqrt{8}}{3}|0\rangle+\frac{1}{3}|2\rangle & \left|\Phi_{b}\right\rangle=\frac{\sqrt{8}}{3}|2\rangle+\frac{1}{3}|-3\rangle \\
\left|\Phi_{c}\right\rangle=\frac{\sqrt{7}}{3}|2\rangle+\frac{\sqrt{2}}{3}|-2\rangle & \left|\Phi_{d}\right\rangle=\frac{\sqrt{7}}{3}|0\rangle+\frac{\sqrt{2}}{3}|-3\rangle \\
\left|\Phi_{e}\right\rangle=\frac{\sqrt{6}}{3}|0\rangle+\frac{\sqrt{3}}{3}|2\rangle & \left|\Phi_{f}\right\rangle=\frac{\sqrt{6}}{3}|2\rangle+\frac{\sqrt{3}}{3}|-3\rangle \\
\left|\Phi_{g}\right\rangle=\frac{\sqrt{5}}{3}|2\rangle+\frac{\sqrt{4}}{3}|-2\rangle & \left|\Phi_{h}\right\rangle=\frac{\sqrt{5}}{3}|0\rangle+\frac{\sqrt{4}}{3}|-3\rangle
\end{array}
$$

1) If you measured the $z$-component of angular momentum, what is the probability that you would obtain $2 \hbar$ ? $-3 \hbar$ ?
2) If you measured the $z$-component of angular momentum, what other possible values could you obtain with non-zero probability?
3) If you measured the energy, what possible values could you obtain with non-zero probability?
4) What is the expectation value of $\hat{L}_{z}$ in this state? the expectation value of energy?
