Central Forces Homework 9

Due 6/7/17, 4 pm

For every problem, before you start the problem, make a brief statement of the form that a correct solution should have, clearly indicating what quantities you need to solve for. This statement will be graded.

PRACTICE:

1. Use the recurrence relation for the radial wave function to construct the $n = 3$ radial states of hydrogen. Calculate the normalization constant for the $R_{32}(r)$ state.

2. By direct application of the differential operators, verify that the state $|321\rangle \doteq \psi_{321}(r, \theta, \phi)$ is an eigenstate of H, $\mathbf{L}^2$, and L_z and determine the corresponding eigenvalues.

REQUIRED:

3. Write out the first 9 terms in the sum:

$$\sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} c_{\ell,m} Y_{\ell,m}$$

   Describe the energy degeneracy of the rigid rotor system, i.e. give the number of eigenstates that all have the same energy.

4. Consider the normalized function:

$$f(\theta, \phi) = \begin{cases} N\left(\frac{\pi^2}{4} - \theta^2\right) & 0 < \theta < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < \theta < \pi \end{cases}$$

   where

$$N = \frac{1}{\sqrt{\frac{\pi^5}{8} + 2\pi^3 - 24\pi^2 + 48\pi}}$$

   (a) Find the $|\ell, m\rangle = |0, 0\rangle$, $|1, -1\rangle$, $|1, 0\rangle$, and $|1, 1\rangle$ terms in a spherical harmonics expansion of $f(\theta, \phi)$.

   (b) If a quantum particle, confined to the surface of a sphere, is in the state above, what is the probability that a measurement of the square of the total angular momentum will yield $2\hbar^2$? $4\hbar^2$?

   (c) If a quantum particle, confined to the surface of a sphere, is in the state above, what is the probability that the particle can be found in the region $0 < \theta < \frac{\pi}{6}$ and $0 < \phi < \frac{\pi}{6}$? Repeat the question for the region $\frac{5\pi}{6} < \theta < \pi$ and $0 < \phi < \frac{\pi}{6}$. Plot your approximation from part (a) above and check to see if your answers seem reasonable.

5. Make a table, similar to the one you made for a particle confined to a ring, showing the different representations of the physical quantities associated with the rigid rotor. Include information about the operators $\hat{H}$, $\hat{L}_z$, and $\hat{L}^2$.

6. Consider the initial state $\frac{1}{\sqrt{2}}\left(|2,0,0\rangle + |2,1,0\rangle\right)$ which is an *sp* hybrid orbital which occurs in chemistry in the study of molecular bonding.

   (a) If you measure the energy of this state, what possible values could you obtain?

   (b) What is this state as a function of time?

   (c) Calculate the expectation value $\langle \hat{L}^2 \rangle$ in this state, as a function of time. Did you expect this answer? Comment.

   (d) Write the time-dependent state in wave function notation.

   (e) Calculate the expectation value $\langle \hat{z} \rangle$ as a function of time. Do you expect this answer?

Rigid Rotor/Particle on a Sphere

	Ket Representation	**Wave Function Representation**	**Matrix Representation**
Hamiltonian			
Eigenvalues of Hamiltonian			
Normalized Eigenstates of Hamiltonian			
Coefficient of the energy eigenstate with quantum numbers ℓ, m			
Probability of measuring $E_{\ell,m}$			

Rigid Rotor/Particle on a Sphere

	Ket Representation	Wave Function Representation	Matrix Representation
Operator for square of the angular momentum			
Eigenvalues of L^2			
Normalized Eigenstates of L^2			
Coefficient of the eigenstates of L^2 with quantum numbers ℓ, m			
Probability of measuring $\hbar^2 \ell(\ell+1)$ for the square of the angular momentum			

Rigid Rotor/Particle on a Sphere

	Ket Representation	**Wave Function Representation**	**Matrix Representation**
Operator for z-component of angular momentum			
Eigenvalues of L_z			
Normalized Eigenstates of L_z			
Coefficient of m^{th} eigenstates of L_z			
Probability of measuring $m\hbar$ for z -component of angular momentum			