Central Forces Homework 8

Due 6/2/17, 4 pm

For every problem, before you start the problem, make a brief statement of the form that a correct solution should have, clearly indicating what quantities you need to solve for. This statement will be graded.

REQUIRED:

1. Use the separation of variables procedure on the angular equation

$$\mathbf{L}^2 Y(\theta, \phi) = A\hbar^2 Y(\theta, \phi)$$

$$\text{where } \mathbf{L}^2 = -\hbar^2 \left[\frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial}{\partial\theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\phi^2} \right]$$

to obtain the following two equations for the polar and azimuthal angles:

$$\left[\frac{1}{\sin\theta} \frac{d}{d\theta} \left(\sin\theta \frac{d}{d\theta} \right) - B \frac{1}{\sin^2\theta} \right] \Theta(\theta) = -A\Theta(\theta)$$

$$\frac{d^2\Phi(\phi)}{d\phi^2} = -B\Phi(\phi)$$

2. **Laguerre Polynomials**
The differential equation for Laguerre polynomials $L_m(z)$ is given by

$$zL'' + (1-z)L' + nL = 0$$

Find a polynomial solution of this differential equation for the case $n = 4$. For what values of z is your solution valid?

3. Show that if a linear combination of ring energy eigenstates is normalized, then the coefficients must satisfy

$$\sum_{m=-\infty}^{\infty} |c_m|^2 = 1$$

4. Answer the following questions for a quantum mechanical particle confined to a ring. You may want to use the Maple worksheet (cfqmring.mw) on time dependence on the ring to help you figure out the answers.

   (a) Characterize the states for which the probability density does not depend on time.

   (b) Characterize the states that are right-moving.

   (c) Characterize the states that are standing waves.

(d) Compare the time dependence of the three states:

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle + |-3\rangle\right)$$
$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle - |-3\rangle\right)$$
$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle + i|-3\rangle\right)$$

5. Consider the following normalized state for the rigid rotor given by:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|1,-1\rangle + \frac{1}{\sqrt{3}}|1,0\rangle + \frac{i}{\sqrt{6}}|0,0\rangle$$

(a) What is the probability that a measurement of L_z will yield $2\hbar$? $-\hbar$? $0\hbar$?

(b) If you measured the z-component of angular momentum to be $-\hbar$, what would the state of the particle be immediately after the measurement is made? $0\hbar$?

(c) What is the expectation value of L_z in this state?

(d) What is the expectation value of L^2 in this state?

(e) What is the expectation value of the energy in this state?

(f) (Challenge:) What is the expectation value of L_y in this state? L_y in spherical coordinates is given by:

$$L_y = i\hbar\left(-\cos\phi\frac{\partial}{\partial\theta} + \cot\theta\sin\phi\frac{\partial}{\partial\phi}\right)$$