Central Forces Homework 5

Due 5/24/17, 4 pm

OPTIONAL:

1. Attached, you will find a copy of the quiz you took yesterday in class. If you are afraid that you did poorly, you can retake the quiz and turn it in for homework, using any resources that you want (including other students) as long as you reference them properly. Make sure that each answer that you turn in is absolutely correct and written up cleanly and carefully. I'm not sure yet how I will incorporate this redo into your grade, but it can't hurt.

PH 426 Quiz—2017

Ground Rules:

No books or other reference materials are allowed, except those provided by the instructor. No calculators are allowed. For **all** problems, show your work or describe your reasoning. Guessing is ok **if** you briefly describe your inspiration—make it believable please. If you need an equation which you can't remember, ask for help.

Some possibly useful equations:

Equations of Motion in Polar Coordinates:

$$\ddot{r} = \frac{1}{\mu} f(r) + \frac{\ell^2}{\mu^2 r^3} \tag{1}$$

$$\dot{\phi} = \frac{\ell}{\mu r^2} \tag{2}$$

Angular Momentum in Polar Coordinates:

$$\ell = \mu r^2 \dot{\phi} \tag{3}$$

Effective Potential:

$$U_{eff} = U(r) + \frac{\ell^2}{2\mu r^2} \tag{4}$$

1. (5+10+5 points) Consider the differential equation:

$$y'' + 2(z+1)y = 0$$

   (a) If you were to find the general power series solution of this differential equation, expanded around $z = -1$, what form would it take?

   (b) Find the first three nonzero terms of a series expanded around $z = -1$ for ONE solution of the differential equation.

   (c) Where does this solution converge?

2. (10 points) Consider the differential equation

$$\frac{d^2y}{dz^2} - \frac{dy}{dz} + Ae^{2z}\,y = 0$$

   Make the change of variables $x = e^z$ in the differential equation above.

3. (5+5+5 points) The spherical harmonic oscillator is defined by the potential $U(r) = \frac{1}{2}kr^2$ in spherical coordinates.

   (a) If a particle in a spherical harmonic oscillator potential begins at rest at $r = R$, draw the resulting effective potential.

   (b) If a particle in a spherical harmonic oscillator potential begins at rest at $r = R$, describe the subsequent motion.

   (c) If the particle is initially at $r = R$, but NOT at rest, what angular momentum would it need to have to be in a circular orbit?