Central Forces Homework 3

Due 5/17/17, 4 pm

For every problem, before you start the problem, make a brief statement of the form that a correct solution should have, clearly indicating what quantities you need to solve for. This statement will be graded.

PRACTICE:

1. Show that the plane polar coordinates we have chosen are equivalent to spherical coordinates if we make the choices:

   (a) The direction of z in spherical coordinates is the same as the direction of $\vec{L}$.

   (b) The θ of spherical coordinates is chosen to be $\pi/2$, so that the orbit is in the equatorial plane of spherical coordinates.

2. Show that the plane of the orbit is perpendicular to the angular momentum vector $\vec{L}$.

REQUIRED:

3. NASA has launched a satellite into a **circular** orbit around the earth and wants to increase the radius slightly while maintaining a circular orbit. NASA scientists propose to fire the engines briefly, applying a small impulse to the satellite.

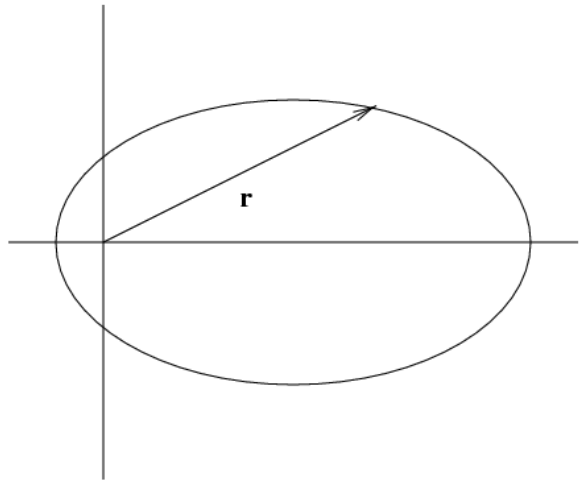
   One scientist says that it doesn't matter if the impulse is applied in a direction tangential to the satellite motion or perpendicular to the motion, arguing that both approaches will simply fine tune the total energy of the system.

   A second scientist disagrees and argues that one of the options would work but the other would definitely not work.

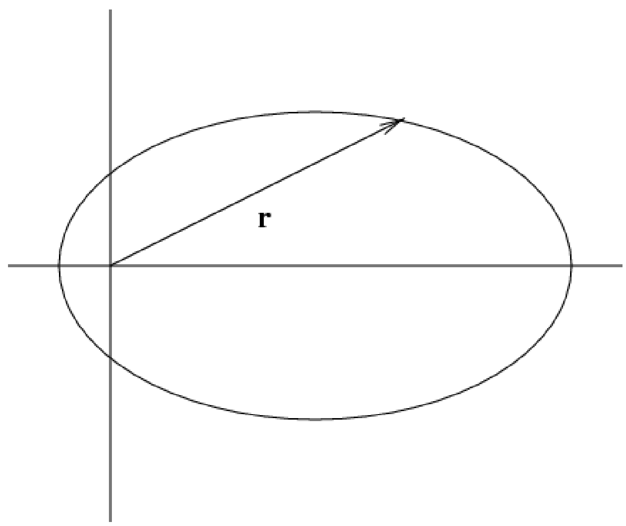
   A third scientist says that neither option would work. Which scientist would you side with, and why?

4. The figure below shows the position vector $\mathbf{r}$ and the orbit of a "fictitious" reduced mass.

   (a) Assuming that $m_2 = m_1$, draw on the figure the position vectors for m_1 and m_2 corresponding to $\mathbf{r}$. Also draw the orbits for m_1 and m_2. Describe a common physics example of central force motion for which $m_1 = m_2$.



(b) Repeat the previous problem for $m_2 = 3m_1$.



5. Consider a system of two particles.

   (a) Show that the total kinetic energy of the system is the same as that of two "fictitious" particles: one of mass $M = m_1 + m_2$ moving with the speed of the CM (center of mass) and one of mass μ (the reduced mass) moving with the speed of the relative position $\vec{r} = \vec{r}_2 - \vec{r}_1$.

   (b) Show that the total angular momentum of the system can be similarly decomposed into the angular momenta of these two fictitious particles.

6. The general equation for a straight line in polar coordinates is given by:

$$r(\phi) = \frac{r_0}{\cos(\phi - \delta)}$$

   Find the polar equation for the following straight lines:

(a) $y = 3$
(b) $x = 3$
(c) $y = -3x + 2$

7. Consider the frictionless motion of a hockey puck of mass m on a perfectly circular bowl-shaped ice rink with radius a. The central region of the bowl ($r < 0.8a$) is perfectly flat and the sides of the ice bowl smoothly rise to a height h at $r = a$.

   (a) Draw a sketch of the potential energy for this system. Set the zero of potential energy at the top of the sides of the bowl.

   (b) Situation 1: the puck is initially moving radially outward from the exact center of the rink. What minimum velocity does the puck need to escape the rink?

   (c) Situation 2: a stationary puck, at a distance $\frac{a}{2}$ from the center of the rink, is hit in such a way that it's initial velocity $\vec{v}_0$ is perpendicular to its position vector as measured from the center of the rink. What is the total energy of the puck immediately after it is struck?

   (d) In situation 2, what is the angular momentum of the puck immediately after it is struck?

   (e) Draw a sketch of the effective potential for situation 2.

   (f) In situation 2, for what minimum value of $\vec{v}_0$ does the puck just escape the rink?

8. In a solid, a free electron doesn't "see" a bare nuclear charge since the nucleus is surrounded by a cloud of other electrons. The nucleus will look like the Coulomb potential close-up, but be "screened" from far away. A common model for such problems is described by the Yukawa or screened potential:

$$U(r) = -\frac{k}{r}e^{-\frac{r}{\alpha}}$$

   (a) Graph the potential, with and without the exponential term. Describe how the Yukawa potential approximates the "real" situation. In particular, describe the role of the parameter α.

   (b) Draw the effective potential for the two choices $\alpha = 10$ and $\alpha = 0.1$ with $k = 1$ and $\ell = 1$. For which value(s) of α is there the possibility of stable circular orbits?