Central Forces Homework 2

Due 5/12/17, 4 pm

For every problem, before you start the problem, make a brief statement of the form that a correct solution should have, clearly indicating what quantities you need to solve for. This statement will be graded.

PRACTICE:

1. If a central force is the only force acting on a system of two masses (i.e. no external forces), what will the motion of the center of mass be?

   The motion of the center of mass will remain unchanged.

2. Which of the following forces can be central forces? which cannot?

   (a) The force on a test mass m in a gravitational field $\vec{g}$, i.e. $m\vec{g}$

   (b) The force on a test charge q in an electric field $\vec{E}$, i.e. $q\vec{E}$

   (c) The force on a test charge q moving at velocity $\vec{v}$ in a magnetic field $\vec{B}$, i.e. $q\vec{v}\times\vec{B}$

3. Using your favorite graphing package, make a plot of the reduced mass μ as a function of m_1 and m_2. What about the shape of this graph tells you something about the physical world that you would like to remember. You should be able to find at least three things.

REQUIRED:

4. Consider the differential equation $z^2y'' + zy' - (1-z)y = 0$. (This is known as Bessel's Equation.)

   (a) Using the change of variable $z = 2\sqrt{u}$, rewrite Bessel's Equation in terms of u.

5. Consider the differential equation $(1-z^2)y'' - 2zy' + l(l+1)y = 0$. (This is known as Legendre's Equation.)

   (a) Use a power series expanded about $z = 0$ to find the first six terms in each of two independent solutions to this differential equation for $l = 2$.

   (b) For what values of z do you expect your power series solutions to converge?

   (c) Find at least one solution to this differential equation for $l = 2$ that does converge outside the range you identified above.

   (d) Using the change of variable $z = \cos\theta$, rewrite Legendre's Equation in terms of θ.

6. Consider two particles of equal mass m. The forces on the particles are $\vec{F}_1 = 0$ and $\vec{F}_2 = F_0\hat{x}$. If the particles are initially at rest at the origin, find the position, velocity, and acceleration of the center of mass as functions of time. Solve this problem in two ways, with or without theorems about the center of mass motion. Write a short description comparing the two solutions.

7. (a) Find $\mathbf{r}_{\rm sun} - \mathbf{r}_{\rm cm}$ and μ for the Sun–Earth system. Compare $\mathbf{r}_{\rm sun} - \mathbf{r}_{\rm cm}$ to the radius of the Sun and to the distance from the Sun to the Earth. Compare μ to the mass of the Sun and the mass of the Earth.

   (b) Repeat the calculation for the Sun–Jupiter system.