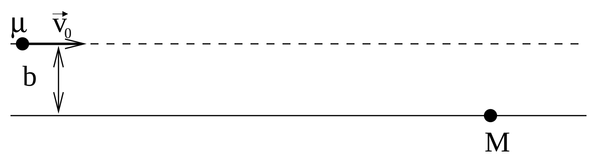
Central Forces Homework 2

Due 2/27/09

REQUIRED:

1. Consider a very light particle of mass μ scattering from a very heavy, stationary particle of mass M. The force between the two particles is a **repulsive** Coulomb force $\frac{k}{r^2}$. The impact parameter b in a scattering problem is defined to be the distance which would be the closest approach if there were no interaction (See Figure). The initial velocity (far from the scattering event) of the mass μ is $\vec{v}_0$. Answer the following questions about this situation in terms of k, M, μ, $\vec{v}_0$, and b. It is not necessarily wise to answer these questions in order.)



Initial Configuration for Problem 5

   (a) What is the initial angular momentum of the system?
   (b) What is the initial total energy of the system?
   (c) What is the distance of closest approach r_{min} (**with** the interaction)?
   (d) Sketch the effective potential.
   (e) What is the angular momentum at r_{min}?
   (f) What is the total energy of the system at r_{min}?
   (g) What is the radial component of the velocity at r_{min}?
   (h) What is the tangential component of the velocity at r_{min}?
   (i) What is the value of the effective potential at r_{min}?
   (j) For what values of the initial total energy are there bound orbits?
   (k) Using your results above, write a short essay describing this type of scattering problem, at a level appropriate to share with another Paradigm student.

More questions on the back

2. The general equation for a straight line in polar coordinates is given by:

$$r(\phi) = \frac{r_0}{\cos(\phi - \delta)}$$

Find the polar equation for the following straight lines:

   (a) $y = 3$
   (b) $x = 3$
   (c) $y = -3x + 2$

3. In a solid, a free electron doesn't "see" a bare nuclear charge since the nucleus is surrounded by a cloud of other electrons. The nucleus will look like the Coulomb potential close-up, but be "screened" from far away. A common model for such problems is described by the Yukawa or screened potential:

$$U(r) = -\frac{k}{r}e^{-\frac{r}{\alpha}}$$

   (a) Graph the potential, with and without the exponential term. Describe how the Yukawa potential approximates the "real" situation. In particular, describe the role of the parameter α.
   (b) Draw the effective potential for the two choices $\alpha = 10$ and $\alpha = 0.1$ with $k = 1$ and $\ell = 1$. For which value(s) of α is there the possibility of stable circular orbits?