PH422: Static Fields

Quiz 3

The quiz on Monday, January 27, will be on the dot and cross products in rectangular, cylindrical, and spherical coordinates. You can find more information at:
`physics.oregonstate.edu/mathbook/GSF/dot.html`
`physics.oregonstate.edu/mathbook/GSF/basisvectors.html`
`physics.oregonstate.edu/mathbook/GSF/orthogonal.html`
`physics.oregonstate.edu/mathbook/GSF/cross2.html`

Some sample types of questions are:

1. Are the following equalities true or false? Why?

   (a)
   $$(\hat{x} - \hat{z}) \cdot \hat{x} = -1$$

   Solution:

   False. The dot product is distributive, so
   $$(\hat{x} - \hat{z}) \cdot \hat{x} = \hat{x} \cdot \hat{x} - \hat{z} \cdot \hat{x} = 1 + 0 = 1$$

   (b)
   $$(\hat{x} - \hat{z}) \times \hat{x} = -\hat{y}$$

   Solution:

   True. The cross product is also distributive, so
   $$(\hat{x} - \hat{z}) \times \hat{x} = \hat{x} \times \hat{x} - \hat{z} \times \hat{x} = 0 - \hat{y},$$
   since $\hat{z} \times \hat{x} = \hat{y}$.

   (c)
   $$(\hat{x} + \hat{y}) \cdot \hat{z} = 0$$

   Solution:

   True. The dot product is distributive and $\hat{z}$ is perpendicular to both $\hat{x}$ and $\hat{y}$.

   (d)
   $$(\hat{x} + \hat{y}) \times \hat{z} = \hat{x} + \hat{y}$$

   Solution:

   False. The cross product is distributive, but not commutative. Thus,
   $$(\hat{x} + \hat{y}) \times \hat{z} = \hat{x} \times \hat{z} + \hat{y} \times \hat{z} = -\hat{y} + \hat{x}.$$

2. Are the following equalities true or false? Why?

(a)
$$\hat{y} \cdot \hat{z} = \hat{x}$$

Solution:

False. $\hat{y}$ and $\hat{z}$ are perpendicular to each other and therefore, the dot product is zero. However, for a right-handed coordinate system, $\hat{y} \times \hat{z} = \hat{x}$.

(b)
$$\hat{s} \times \hat{z} = -\hat{\phi}$$

Solution:

True. In a cylindrical coordinate system, $\hat{s}$, $\hat{\phi}$, and $\hat{z}$ are a right-handed triple (like $\hat{x}$, $\hat{y}$, and $\hat{z}$).

(c)
$$\hat{r} \times \hat{\phi} = \hat{\theta}$$

Solution:

False. In spherical coordinates, $\hat{r}$, $\hat{\theta}$, and $\hat{\phi}$ are the right-handed triple. Thus, $\hat{r} \times \hat{\phi} = -\hat{\theta}$.

(d)
$$\hat{r} \cdot \hat{\theta} = 1$$

Solution:

False. $\hat{r}$ and $\hat{\theta}$ are perpendicular to each other and thus, the dot product is zero.