The formats (type, length, scope) of these Prep problems have been purposely created to closely parallel those of a typical exam (indeed, these problems have been taken from past exams). To get an idea of how best to approach various problem types (there are three basic types), refer to these sample problems.
1. Evaluate each statement (T/F/N). *As always you must fully and correctly explain your answer and reasoning.*

   a. (i) A moving charge causes only a magnetic field.
      False. Any charge, no matter whether it’s moving or not, causes an electric field.
   
      (ii) Magnetic field lines always form closed loops.
      True.
   
      (iii) Magnetic field lines are always perpendicular to any forces they exert.
   
      (iv) A magnetic field can accelerate a moving charge.
   
      (v) All magnetic field lines caused by the same current are always parallel to one another.
      False. Suppose the current is not in a straight wire—it’s curved or bent in some random fashion.

   b. (i) If east is ↔, and north is X, then down is ↑.
   
      (ii) If up is ↔, and north is ↑, then west is X.
   
   (iii) A charged particle moving at constant speed must cross the lines of a magnetic field at an angle of 45° in order to feel half the maximum force that field could exert on the particle.
      False. \( F_{mag} = qvB \sin q \). Therefore the maximum force magnitude is simply \( qvB \), so half that magnitude would be exerted when \( \sin q = 0.5 \). That’s an angle of 30°, not 45°.
   
   (iv) If two charged particles are identical except for charge (+e and +2e), and both are moving in the same perpendicular magnetic field at the same speed, the +e charge will move in a smaller circle.
      False. The radius of the circular path of a charged particle moving in a perpendicular magnetic field is given by \( r = \frac{mv}{(qB)} \), so the larger charge, 2e, would move in the smaller circle.

   c. (i) An electron moving parallel to a current-carrying wire has no magnetic force acting on it.
      False. The field from the wire is perpendicular to that wire, which (because the electron’s path is parallel to the wire) is also perpendicular to the electron’s path.
   
      (ii) If a straight segment of current-carrying wire is pushed to the north by a magnetic field, and the current is flowing westward, then the field must be vertically upward.
   
   (iii) If an electric motor’s loop is carrying current clockwise (as you view it), and the magnet’s field is directed from right to left, the loop’s left side will be pushed out toward you.
      True. Use the right-hand rule for force to test this: Right thumb upward (because, in your view, the left side of a clockwise current loop is flowing upward), right fingers to the left; indeed, your right palm is facing you.
   
   (iv) An electric motor’s torque is at maximum when its loop(s) of wire have no magnetic field lines going through them.
      True. Torque is a product of force and lever arm, but it is attenuated by the sine of the angle between them. And while the magnetic force on each segment of the current loop doesn’t change, nor does the lever arm (the distance from the force application point to the axis of rotation), the angle between them varies from 90° to 0°. In the case described, it’s 90°, and \( \sin 90° \) allows the maximum torque.
   
   (v) Every current loop is a magnet.
      True. Any moving charge produces a magnetic field.

   (vi) Anti-parallel current loops repel each other.
      True. First, study Fig. 32.47b on page 947 (anti-parallel straight lines of current), then realize that any current loop can be modeled as many short connected segments of straight wire, producing the results shown in Fig. 32.49a on page 948.
1. d. (i) A proton that initially moves horizontally eastward here in Corvallis would gain altitude as a result of being deflected by the earth’s magnetic field.
   True. Use the right-hand rule for force. The earth’s magnetic field here in corvallis is oriented about 16° east of geographic north, but tilted about 67° downward. With a flat right hand, aim your fingers in that direction, then point your thumb horizontally eastward. Your palm will then be facing in a direction whose vertical component is indeed upward — sufficient to cause the proton to gain altitude.

(ii) Magnetic field lines emerge from the earth at geographic south.
   True. They do emerge, but at a non-perpendicular angle. Magnetic field lines emerge vertically from the earth only at magnetic south.

(iii) If we model a bar magnet as electrons in tiny but aligned current loops (“spins”), then as you face the south end of the bar magnet, the “spin direction” of the electrons is counter-clockwise (ccw).
   True. The south end of the bar magnet is, by definition, where field lines enter the magnet (i.e. dive into it — away from you, from your view). Negative charge going in a ccw loop is equivalent to positive charge going in a cw loop. Test that with RHR #2 (the field rule): With your right thumb pointing in the cw direction of a current loop (easiest at the bottom in this case), your fingers do indeed “dive into” the middle of that loop.

(iv) If you break a compass needle in half, either half will correctly operate as a compass needle.
   True. Each half will be a magnet with the same polarity as the original; every magnet is composed of many small field-aligned regions called magnetic domains, and there are still plenty of those regions in either half of the broken compass needle.

e. An electron moves at constant velocity under the influence of both electric and magnetic fields. Evaluate each statement (T/F/N). As always, you must fully and correctly explain your answer and reasoning.

(i) The fields are parallel (point in the same direction).
   False. In order for the electron to be in equilibrium (travel at constant velocity), the two forces, \( F_E \) and \( F_{\text{mag}} \), must oppose (be anti-parallel). But \( F_{\text{mag}} \) is always perpendicular to \( B \), while \( F_E \) is always aligned with (parallel or anti-parallel to) \( E \). So \( E \) must be perpendicular to \( B \).

(ii) The fields are perpendicular. True. See reasoning in (i), above.

(iii) The fields are anti-parallel (opposite directions). False. See reasoning in (i), above.

f. A long, straight current, \( I \), exists along a vertical path through the origin, upward out of the earth. With the \( x-y-z \) coordinate system shown (and \( \angle \) notation for the \( x-y \) plane only), evaluate each statement (T/F/N). Disregard the earth’s B-field. As always, explain your reasoning.

(i) All magnetic field loops here are circular and concentric.
   False. The are all circular and their centers are all along one line, but they do not all actually share the same center point (which is the definition of concentric).

(ii) The point \((2, -2, 0)\) has a magnetic field direction of \( \angle -45^\circ \).
   False. By RHR #2, the field direction there is +45°. (The point in question is in the plane of the paper, at a position angle of \(-45^\circ\), but the field there is tangent to that circle — in the counter-clockwise direction.)

(iii) All points along the negative \( x \)-axis have the same \( B \) field value.
   False. The field value decreases with distance from the current (which is along the \( z \)-axis).

g. As the diagram indicates, \( I_1 \) is anti-parallel to \( I_2 \), and each current-carrying wire has the length shown. Evaluate each statement (T/F/N). As always, you must fully and correctly explain your reasoning.

(i) \(|F_{\text{mag},1}| = I_1 L_1 B_1 \sin 90^\circ|.
   True. Since \( L_1 > L_2 \), the only portion of \( I_1 \) that experiences field lines produced by \( I_1 \) is a length equal to \( L_1 \). That is, the effective length of \( I_1 \) (for the purposes of receiving force exerted by \( I_1 \) is \( L_1 \), not \( L_2 \).
   Thus (by Newton’s Third Law): \(|F_{\text{mag},2}| = |F_{\text{mag},1}| = I_1 L_1 B_1 \sin 90^\circ|.

(ii) If \( I_1 = I_2 \), the \( B \) field value midway between the wires is zero.
   False. By RHR #2 (the field rule), the two fields are parallel at that midpoint, not antiparallel.

(iii) These two wires repel one another.
   True. By RHR #2, for example, \( B_1 \) is directed out of the page at the set of points where \( I_2 \) is flowing. Then by RHR #1, the force on \( I_1 \) is down the page — away from \( I_1 \).
1. **h.** Two identical wires are carrying equal but anti-parallel currents. The wires rest horizontally on the earth, as shown in this overhead view. For this problem, disregard any magnetic field caused by the earth.

Evaluate (T/F/N) each statement. As always, you must justify your answers fully with any mix of valid explanations, drawings and calculations.

(i) If an electron is traveling south through a point that is 1 m directly (vertically) above \( I_1 \), the net magnetic force on it will be horizontal.
   
   **False.** At the point in question (by RHR for fields): \( B_1 \) is directed horizontally due west; \( B_2 \) has a horizontal component due east, but also a vertical component upward. And (because B is weaker at a greater distance from a current): \(|B_1| > |B_2|\)  
   So \( B_{\text{net}} \) will have a westward horizontal component and an upward vertical component. Thus, (by RHR for forces), the magnetic force on the electron moving due south will have an eastward horizontal component and an upward vertical component (i.e. same quadrant as \( B_2 \)).

(ii) If a proton is traveling downward through point P, which is midway between the wires, the net magnetic force on it will be zero. (Disregard the fact that it’s about to hit the ground, too.)
   
   **True.** For a point charge \( q \) moving at speed \( v \) and angle \( \theta \) through a magnetic field \( B_{\text{net}} \): \(|F_{\text{net}}| = qvB_{\text{net}} \sin \theta\)
   At point P (by the RHR for fields), the two field directions are parallel (they do not cancel); \( B_{\text{net}} \neq 0\)
   However, the direction of \( B_{\text{net}} \) is anti-parallel to the proton’s velocity (so \( \theta = 180^\circ \), and \( \sin 180^\circ = 0 \)).
   Therefore: \(|F_{\text{net}}| = qvB \sin 0 = 0\)

(iii) The net magnetic force on \( I_2 \) is directed to the east.
   
   **True.** Anti-parallel currents exert repelling magnetic forces on one another.
2. a. Express the units of \( \mu_0 \) in only basic SI units (kg, m, s and C). Simplify/reduce this as much as possible.
\[
B = \mu_0 J/(2\pi r), \text{ so } \mu_0 \text{ units are } \text{T} \cdot \text{m} / \text{A} = \frac{\text{N} \cdot \text{s}}{\text{C} \cdot \text{m}} \cdot \frac{\text{m}}{\text{s}} = \frac{\text{N} \cdot \text{s}^2}{\text{C}^2} = \text{kg} \cdot \text{m} / \text{C}^2
\]

b. Suppose a region of space has both an E-field and a B-field. Express the ratio of E-field strength to B-field strength \((E/B)\) in fundamental (“base”) SI units (kg, m, s and C). Simplify or reduce as far as possible.
\[
E/B \text{ has units of } \frac{\text{N}/\text{C}}{\text{T}}, \text{ but Teslas (T) are } \frac{\text{N}}{\text{C} \cdot \text{m/s}}. \text{ So } E/B \text{ has units of } \frac{\text{m/s}}{\text{s}} = \text{m/s}. 
\]

c. Give four different reasons why a particle would not be affected by a magnetic field.
It’s not a charged particle; or it’s not in the magnetic field; or it’s not moving; or it’s moving, but only parallel or anti-parallel to the magnetic field.

d. One of the Right-Hand Rules determines the direction of magnetic force; the other Right-Hand Rule determines the direction of magnetic field.

e. In a copper wire electrons move in the horizontal plane (neither up nor down) and to your left. The direction of the magnetic field of a large magnet is toward you. In which direction is the magnetic force on the electrons in the copper wire caused by the magnet? **Downward.** Point your right thumb to your left and your right fingers toward you. Then the back of your hand will be facing downward.

f. If a stream of electrons is traveling vertically upward, what is the direction of the magnetic field at a position 1 cm east of that current?
A current of negative charge traveling vertically upward is equivalent to a current of positive charge traveling vertically downward. So, using RHR #2 (the field rule), with your right thumb pointing vertically down, your fingers will curl in the direction of the field. east of your thumb, your fingers will be directed to the south.

g. A proton traveling horizontally northward will turn first in what direction if it enters a uniform magnetic field that is directed to the west? **Upward.** (Right thumb north; right fingers west; right palm will face upward.)

h. There are at least three ways to change the magnetic properties of a ferromagnet. Describe each way.
Place the ferromagnet into a larger magnetic field that is oriented differently than the ferromagnet.
Raise the temperature of the ferromagnet until the additional thermal energy effectively disrupts (randomizes) the alignment of the local magnetic domains.
Hammer on the ferromagnet until the additional mechanical energy (vibration) effectively disrupts (randomizes) the alignment of the local magnetic domains.

i. Shown here are four steel bars. Three are magnets. One of the poles is indicated. Via experiment, we find that ends \(a\) and \(d\) attract each other; ends \(c\) and \(f\) repel; ends \(e\) and \(h\) attract; and ends \(a\) and \(h\) attract.

Which bar is not a magnet?

The fourth bar \((g-h)\). The second and third bars must both be magnets, because a magnet will always attract a ferrous non-magnet, never repel it (because the field from the magnet causes the magnetic domains in the non-magnet to (temporarily) become parallel (not anti-parallel) to that field.

j. Protons accelerated through a potential difference of 5.00 MV are moving in the \(+x\)-direction. Suddenly they enter a magnetic field of magnitude 0.03 T in the positive \(z\)-direction. The field extends from \(x = 0\) to \(x = 1.00\) m. Calculate the \(y\)-component of the protons’ momentum as they leave the magnetic field.
3. a. An electron moves with uniform circular motion in a mass spectrometer. The strength of the magnetic field is 25.0 mT, and the magnitude of its momentum is $1.25 \times 10^{-23}$ kg·m/s. What is the magnitude of its acceleration?

$$a = \frac{v^2}{r}, \text{ where } r = \frac{mv}{qB}$$

But $mv = p$, or $v^2 = \frac{p^2}{m^2}$

Thus: $a = \frac{(p^2/m^2)/[p/(qB)]}{p/m} = \frac{pqB}{m^2} = \frac{(1.25 \times 10^{-23})(1.60 \times 10^{-19})(0.025)}{(9.11 \times 10^{-31})^2} = 6.02 \times 10^{16}$ m/s$^2$

b. You send two identical charged particles (A and B) into the same magnetic field, and they both move in circles within that field. However, $v_B = 3v_A$. Write an equation, in the form of $T_B = ( )^{( )} T_A$, to relate the periods of their motions.

$$T = \frac{2\pi r}{v} \quad \text{and} \quad r = \frac{mv}{|qB|}$$

So: $T_B/T_A = \frac{[2\pi m_A/(q_A B_A)]/[2\pi m_A/(q_A B_A)]}{1} = 1$ (because the particles are identical and in the same field).

Thus: $T_B = T_A$

c. At a certain moment in time, an electron is traveling due south. Then, at another moment in time, just $1.75 \times 10^6$ s later, that electron is traveling due west. There is a steady, uniform magnetic field in this region, directed vertically upward. What minimum strength must that field have? (Ignore gravity and air.)

d. Gaseous hydrogen (H$_2$) is partly ionized, giving a mix of five particle types:

- H$_2$ (two protons, two electrons)
- H$_2^+$ (two protons, one electron)
- H (one proton, one electron)
- H$^+$ (single proton)
- e$^-$ (single electron)

The mix of particles is sent into a uniform, perpendicular magnetic field ($B = 3.21$ mT). Their paths are shown as dotted lines; their collectors (□) have their entrances positioned as follows (lengths in mm):

#1: (-14, 0)  #2: (-10, 0)  #3: (0, 20)  #4: (5, 0)

(i) Collector #2 is for single protons. Match the other paths with the particle(s) traveling them.

(ii) For each particle traveling a curved path, calculate its speed. (You’ll need to look up various masses.)

(i) An uncharged particle will move through a magnetic field with no magnetic force acting upon it.

So the H$_2$ and H particles follow the linear path to Collector #3.

But a charged particle moving in a perpendicular B-field follows a circular path. And, by RHR #1, if the H$^+$ particle (the proton) turns counter-clockwise, the B-field here is directed into the page (X). Therefore...

The electron must be turning right, traveling to Collector #4.

And the other positive particle (H$_2^+$) must be traveling to Collector #1.

(ii) Each charged particle’s path has a radius of $r = \frac{mv}{|qB|}$. Thus: $v = \frac{r|q||B|}{m}$

$$v_1 = r_1|q||B|/m_1 = (0.007)(1.60 \times 10^{-19})(3.21 \times 10^3)/(2(1.67 \times 10^{-27}) + 9.11 \times 10^{-31}) = 1.08 \times 10^3 \text{ m/s}$$

$$v_2 = r_2|q||B|/m_2 = (0.005)(1.60 \times 10^{-19})(3.21 \times 10^3)/1.67 \times 10^{-27} = 1.54 \times 10^3 \text{ m/s}$$

$$v_4 = r_4|q||B|/m_4 = (0.0025)(1.60 \times 10^{-19})(3.21 \times 10^3)/9.11 \times 10^{-31} = 1.41 \times 10^6 \text{ m/s}$$
3. e. One type of particle has mass 4 x 10^{-26} kg, charge +2e, and speed 1000 m/s. Another type of particle has mass of 5 x 10^{-26} kg, charge –3e, and speed 913 m/s. A mixed stream of these two particle types is sent eastward along the x-axis into a region (shaded) where a uniform vertical magnetic field, B, exists. Two collectors, each facing (opening toward) the east, must be placed along the y-axis to catch the particles as they exit the field. One collector is correctly positioned as shown, at y = 10.0 cm. Where (along the y-axis) must the other collector be positioned?

Any charged particle that enters a perpendicular B-field will thereupon move in a circular path with a radius given by: \( r = \frac{mv}{|q|B} \)

By RHR #1, the positively charged particle will turn in a counter-clockwise sense (in the view given) and thus arrive at the collector shown, after completing a semi-circle. So the radius of its orbit is: \( r_1 = \frac{mv_1}{|q_1|B} \)

And the y-distance of its collector from origin is the diameter, \( d_1 \), twice the radius: \( d_1 = 2r_1 = 2\left(\frac{mv_1}{|q_1|B}\right) \)

Likewise, by RHR #1, the negatively charged particle (type 2) will turn in a clockwise sense and thus arrive at some negative y-position, after completing a semi-circle. The radius of its orbit is: \( r_2 = \frac{mv_2}{|q_2|B} \)

And the y-distance of its collector from origin is the diameter, \( d_2 \), twice the radius: \( d_2 = 2r_2 = 2\left(\frac{mv_2}{|q_2|B}\right) \)

Find the ratio \( d_2/d_1: \)

\[
\begin{align*}
d_2/d_1 &= \frac{2m_2v_2/|q_2|B}{2m_1v_1/|q_1|B} \\
&= \frac{m_2v_2^2|q_2|}{m_1v_1^2|q_1|} \\
&= \left(\frac{5\times10^{-26}}{4\times10^{-26}}\right)^2 \times \left(\frac{9.11\times10^{-31}}{1.60\times10^{-19}}\right)^2 \times \left(1.1\times10^6/1.1\times10^6\right)^2 \\
&= 0.7608 \\
\end{align*}
\]

So the position of the second collector should be at \( y = -7.61 \text{ cm} \) (or \( y = -0.0761 \text{ m} \))

f. Two uniform and anti-parallel magnetic fields, \( B_1 \) and \( B_2 \), exist in regions adjacent to one another, as shown here (with coordinate axes —in mm—placed for reference). An electron, traveling eastward at a speed of 966 m/s, enters \( B_1 \) at the point (0, 5). Then sometime later, it exits \( B_1 \) at the point (30, 5).

(i) If the magnitude of \( B_1 \) is 1.10 \( \mu \text{T} \), what is the minimum magnitude of \( B_2? \)

(ii) What is the next least value of \( B_2? \)

(iii) Assuming the minimum value for \( B_2 \), how much time elapses as the electron travels from (0, 5) to (30, 5)?

(i) A magnetic field cannot do work, so the electron’s speed does not change. And \( r = \frac{mv}{|q|B} \).

Find \( r_1 = \frac{mv_1}{|q_1|B_1} = (9.11 \times 10^{-31})(966)/((1.60 \times 10^{-19})(1.1 \times 10^6)) = 5.000 \times 10^{-3} \text{ m} \)

Since this radius (5 mm) is the same as the y-component of the electron’s entry point into \( B_1 \), the electron makes just a quarter-circle path in \( B_1 \) before exiting (directly down the page) into \( B_2 \). And, under the same reasoning, the electron’s final path in \( B_2 \) must also be a quarter circle of radius 5 mm. Therefore, the electron’s final exit point into \( B_2 \) is (5, 0), and it’s traveling directly down the page at that point.

Likewise, the electron’s exit point out of \( B_2 \) is (25, 0), and it’s traveling up the page at that point.

Thus: In the minimum field strength \( B_2 \), the electron completes a semi-circle of diameter 20 mm (\( r = 10 \text{ mm} \)).

Thus: \( r_2/r_1 = \frac{mv_2}{|q_2|B_2}/\frac{mv_1}{|q_1|B_1} = B_2/B_1 = 2 \)

Or: \( B_2 = B_1/2 = (1.1 \times 10^6)/2 = 5.50 \times 10^{-7} \text{ T} \)

(ii) In the next minimum field strength \( B_2 \), the electron would make a semi-circle in \( B_2 \), re-enter \( B_1 \), make a semi-circle there, re-enter \( B_2 \), complete another semi-circle there, then finally exit to \( B_2 \) and arc east and out.

In a horizontal distance of 30 mm, that’s two quarter arcs and one semi-circle, all of radius 5 mm (total: 20 mm), plus two semi-circles of radius \( r_2 \). Thus \( 4r_2 = 10 \text{ mm} \), or \( r_2 = 2.5 \text{ mm} \).

Thus: \( r_2/r_1 = B_2/B_1 = 1/2 \) Or: \( B_2 = 2B_1 = 2(1.1 \times 10^6) = 2.20 \times 10^6 \text{ T} \)

(iii) The total time for the trip from (0, 5) to (30, 5) is the sum of two half-periods—for the circular paths that were partly completed in each field: \( \Delta t_{\text{total}} = T_1/4 + T_2/2 + T_2/4 = T_1/2 + T_2/2 = (T_1 + T_2)/2 \)

\( T_1 = 2\pi r_1/v \) and \( T_2 = 2\pi r_2/v \) Thus: \( \Delta t_{\text{total}} = (T_1 + T_2)/2 = (2\pi r_1/v + 2\pi r_2/v)/2 = (\pi/v)(r_1 + r_2) = (\pi/966)(.005 + .010) = 4.88 \times 10^{-5} \text{ s} \)
3. g. Parallel plates with capacitance \( C \), charge \( Q \), and plate separation \( x \) are located just inside a uniform magnetic field, \( B \) (the shaded area shown), which is directed vertically upward. Protons of various speeds are sent in a stream directly southward on the path shown between the plates. Some of those protons continue on to strike a detection film perpendicularly, as shown. The detector film is placed at an angle of 60° west of north. When such a proton enters the capacitor, it takes 0.500 ms to reach the film (and it travels within the magnetic field for that entire trip). What distance does it travel within the capacitor?

Data: \( C = 3.00 \text{ mF} \quad Q = 640 \text{ nC} \quad x = 1.00 \text{ cm} \quad B = 127 \text{ \mu T} \)
4. a. At what angle must you orient a straight current-carrying wire relative to a magnetic field so that the force on the wire is half the maximum possible force?

\[ F_{\text{mag}} = ILB \sin \theta \]. Therefore the maximum force magnitude is simply \( ILB \), so half that magnitude would be exerted when \( \sin \theta = 0.5 \). That’s when \( \theta = 30^\circ \).

b. An electric motor is a device which converts electrical potential energy into kinetic energy.

c. What is the orientation of the coil (its normal vector) with respect to the magnetic field in an electric motor when the motor achieves zero torque?
The coil’s normal is aligned with (parallel or anti-parallel to) the field. See item 1c(iv).

d. A certain point P in space is affected by two different uniform magnetic fields. One field is 3.00 T, directed to the east. The other field is 5.00 T, directed to the northwest (that’s midway between north and west). Find the total magnetic field (magnitude and direction) at point P.

*This is just vector addition* (indeed the two vectors could have been of any of the same type—positions, velocities, accelerations, momenta, forces, E-fields, etc.). In this case, you’re adding B-fields:

\[ 3 \angle 0^\circ \mathrm{T} + 5 \angle 135^\circ \mathrm{T} = [3 \cos 0^\circ, 3 \sin 0^\circ] \mathrm{T} + [5 \cos 135^\circ, 5 \sin 135^\circ] \mathrm{T} = [-0.5355, 3.5355] \mathrm{T} \]

The magnitude of the resultant vector is:

\[ |B_{\text{total}}| = [(-0.5355)^2 + (3.5355)^2]^{1/2} \mathrm{T} = 3.58 \mathrm{T} \]

The angle of the resultant vector:

\[ \theta = \arctan[3.5355/(-0.5355)] = 98.6^\circ \] (this is in Quadrant II; thus \( \theta > 90^\circ \))

Therefore:

\[ B_{\text{total}} = 3.58 \angle 98.6^\circ \mathrm{T} \]

e. Near (but not at) the north/south geographic pole, the earth’s magnetic field lines are generally directed at some angle into/out of the earth.

f. A circular loop of wire is oriented so that its normal is perpendicular to the earth’s horizontal magnetic field (\( B = 2.87 \times 10^{-5} \mathrm{T} \)). A correctly operating compass is placed at the center of this loop. When a current of 9.54 A is sent through the loop, the compass needle rotates by 60.0°. Calculate the radius of the loop.

g. Viewing a solenoid from one end, if a steady current is flowing through the loops counterclockwise, how will a compass needle respond when placed near that end?
The solenoid end you’re viewing is the north pole of the solenoid field (test this with the field RHR), so the compass needle (its colored North pole) will point away from the solenoid.

h. If you’re looking into the south pole of a magnet formed by a solenoid, which way (from your point of view) is the current flowing around its loops?

From your point of view, the field lines are “diving” into the solenoid loops (the definition of its south pole), which means the current loops are clockwise to you (test this with the field RHR).

i. If you’re initially standing 1.50 m from a long, straight current-carrying wire, what minimum distance do you need to move in order to arrive at a point where there is twice the magnetic field strength of your initial location?

\[ |B_i| = \mu_0 I/(2\pi r_i) \quad \text{and} \quad |B_f| = \mu_0 I/(2\pi r_f) \quad \text{and} \quad |B_f|/|B_i| = 2 \]

Thus:

\[ [\mu_0 I/(2\pi r_f)]/[\mu_0 I/(2\pi r_i)] = 2 \]

Simplified: \( r_i/r_f = 2 \)

Or: \( r_f = r_i/2 = 1.50/2 = 0.75 \text{ m} \)

So: \( r_i - r_f = 1.50 - 0.75 = 0.750 \text{ m} \)
4. j. Two long, parallel wires are each carrying different currents. An electron is moving at a constant velocity along a path between these wires (shown by the dotted line). Find the magnitude and direction of the magnetic field, \( B_p \) at the point \( P \), which is midway between the two wires.

There are no numbers given for this problem, so just express \( B_p \) in terms of \( I_1, d \) and \( D \). You may ignore any field effects caused by the lone electron.

k. Magnetohydrodynamic (MHD) propulsion moves a boat through seawater very efficiently (low turbulence and drag). Below are schematics of a MHD propulsion tube. In the first diagram, seawater enters on the right end of the tube and is pushed out the left end, thus propelling the tube (and thus the boat) to the right.

The key features are a set of long, charged parallel plates and also the opposing poles of a powerful magnet. Each set is mounted along opposite sides of the length of the tube — see the second diagram here, which is a view of the tube looking into the end where the water is expelled.

(i) Using the second diagram, draw/describe how MHD propulsion works. Be sure to indicate the polarity of the plates and magnet along with the direction of any fields and/or currents.

See diagram the here. The E-field (directed left to right here) drives a current in that direction in the (readily electrolytic) saltwater. The B-field (directed up the page here) then pushes this current-carrying water out the tube (out of the page here), as the RHR for force will confirm.

(ii) What change in the setup is necessary for a “reverse gear” — so that the boat can “back up?”

Just reverse the polarity of the charged plates (and thus the E-field). (Reversing the magnetic field would work, too, but it’s less practical.)
5. a. There is a uniform magnetic field of strength \( B_1 \) (created by some external cause) oriented into the page in the region shown on the right of the diagram below. Two current-carrying wires, parallel to one another and separated by a distance \( d \), pass through that region. One of those wires is connected to a battery, located as shown. That circuit has a total resistance \( R \). The other wire is a long, straight wire. There is no net magnetic force on the long straight wire. (The wire section containing the battery and resistor is so far away from the long straight wire that you can ignore any field effects from it.)

What is the voltage, \( V \), of the battery—and which way is it connected? There are no numbers given for this problem, so just express \( V \) in terms of the following known values: \( B_1 \), \( d \) and \( R \).

In the vicinity of the long, straight wire, there are three sources of magnetic fields:

(i) the external cause of \( B_1 \);
(ii) the field caused by the current \( I \) in the long straight wire;
(iii) the field caused by the current in the circuit (call that current \( I_C \) and its field \( B_C \)—and again, note that only the right end of the circuit produces any field of interest here).

However, there is no net magnetic force on the long wire. Since that \( I \)'s own B-field cannot exert any force on itself, this zero net force is the result of the vector sum of the other two fields, \( B_1 \) and \( B_C \).

That is:
\[
B_1 + B_C = 0
\]

So their magnitudes are equal:
\[
B_1 = B_C
\]

And their directions are opposite. Thus \( B_C \) must be directed out of the page along the path of the long straight wire—and by RHR \#2, therefore, \( I_C \) must be clockwise around the circuit (so the battery should be connected with its positive terminal uppermost in the diagram).

To find the voltage, \( V \), of the battery, note that the magnitude of \( B_C \) along the path of the long wire is:
\[
B_C = \mu_0 I_C/(2\pi d)
\]

But \( B_1 = B_C \), so:
\[
B_1 = \mu_0 I_C/(2\pi d)
\]

And \( I_C = V/R \)

Substituting, we get this:
\[
B_1 = \mu_0 (V/R)/(2\pi d)
\]

Rearrange to solve for \( V \):
\[
(2\pi d)B_1 = \mu_0 (V/R)
\]
\[
(2\pi d)B_1 R/\mu_0 = V
\]

In order to have no net magnetic force on the long straight wire, the battery voltage would need to be \((2\pi d)B_1 R/\mu_0\), and the battery would need to be connected with its positive terminal uppermost in the diagram; the current in the circuit flows clockwise.
5. **b.** Two straight wires are placed flat on the earth, in parallel to each other, so that their ends point to the east and west, as shown. Wire 1 has a length of 8.74 m. Wire 2 has a length of 6.93 m. Wire 1 carries a current $I_1$, of 64.0 A, flowing as shown. Wire 2 also carries a current, $I_2$, but neither its magnitude nor its direction is known. Wire 1 is pulling on Wire 2 with a force of 0.598 N. If an electron were to pass through point P while traveling directly south at 312 m/s, what magnetic force (magnitude and direction) would Wire 2 exert on the electron at point P?

![](coordinate_system.png)

1.25 cm

2.17 cm

24.0°

•

I_1

I_2

1.25 cm

2.17 cm

24.0°

P


5. **c.** Evaluate (T/F/N) the following statement. Justify your answer fully with any valid mix of words, drawings and calculations. A straight length of wire is carrying current within a uniform magnetic field. After a bit of experimenting, you find that the maximum magnetic force magnitude that is exerted on the wire is 4 times the force being exerted in its original position. Thus, you had to rotate it by about 75.5° from its original position to the position of maximum force.

**True.** For a straight length $L$ of current $I$ flowing at an angle $\theta$ through a magnetic field $B_{net}$:

$$|F_{net}| = ILB_{net}\sin\theta$$

Thus:

$$|F_{net,\text{max}}| = ILB_{net}\sin\theta_{F,\text{max}} = ILB_{net}\sin90°$$

$$\theta_{F,\text{max}} = 90°$$

We are told that $|F_{net,\text{max}}|/|F_{net,\text{original}}| = 4$ That is: $(ILB_{net}\sin90°)/(ILB_{net}\sin\theta_{\text{original}}) = 4$

Thus: $\sin\theta_{\text{original}} = 1/4$ So: $(ILB_{net}\sin90°)/(ILB_{net}\sin\theta_{\text{original}}) = 14.5°$ This is 75.5° from $\theta_{F,\text{max}}$
5. d. A long straight wire carries a steady current, \( I_1 \), in the direction shown. A distance \( d_1 \) away is the nearest edge of a circular loop of wire that carries a steady current, \( I_2 \), in the direction shown. The loop’s diameter is \( d_2 \). For parts (i) and (ii), assume that the known values are \( I_1, d_1, I_2, d_2 \) and \( \mu_0 \).

(i) Where, in the region enclosed by the dotted box, must the net magnetic field be downward? *Answer with a brief sketch or a complete description.*

(ii) Find an expression for the magnitude of the total magnetic field, \( B_{T,\text{center}} \), at the center of the loop.

(iii) For the direction of the total field, \( B_{T,\text{center}} \), there are three possibilities. List each possibility, along with the condition necessary in each case.

(iv) Assume that the single loop is now \( N \) concentric identical loops, each carrying the current \( I_2 \) (in the same direction as before), and the known values are now \( N, I_1, d_1, d_2 \) and \( \mu_0 \). Also, assume that \( B_{T,\text{center}} = 0 \). Find an expression for \( I_2 \).
5. e. A rectangular loop of wire, with a width $d_1$ (as shown), carries a current, $I_1$. A distance $d_3$ away from the eastern edge of the rectangular loop is the nearest edge of a circular loop of wire that carries a current, $I_2$, in the direction shown. Both currents are steady (unchanging). The circular loop’s diameter is $d_2$. The total magnetic field at the center of the circular loop is zero. Assume that the other known values are $d_1$, $d_3$, $I_2$, $d_2$ and $\mu_0$.

(i) Determine the direction of the current $I_1$.

(ii) Find an expression for the magnitude of $I_1$.

(i) The total magnetic field at the center of the circular loop, $B_{T,\text{center}}$, is the sum of three fields:

$$ B_{T,\text{center}} = B_{1,\text{near,center}} + B_{1,\text{far,center}} + B_{2,\text{center}} = 0 $$

where:

- $B_{1,\text{near,center}}$ is the field at the center of the circular loop caused by the nearer long side of $I_1$
- $B_{1,\text{far,center}}$ is the field at the center of the circular loop caused by the farther long side of $I_1$
- $B_{2,\text{center}}$ is the field at the center of the circular loop caused by $I_2$

By RHR #2, the direction of field $B_{2,\text{center}}$ is downward (i.e. $X$, or into the page). Therefore, the sum of $B_{1,\text{near,center}}$ and $B_{1,\text{far,center}}$ must be upward (i.e. $\cdot$, or out of the page).

Also by RHR #2, the directions of $B_{1,\text{near,center}}$ and $B_{1,\text{far,center}}$ must be opposite to one another. But $|B_{1,\text{near,center}}| > |B_{1,\text{far,center}}|$ because $|B| = \mu_0 I/(2\pi r)$ Therefore, the direction of $B_{1,\text{near,center}}$ must upward (i.e. $\cdot$, or out of the page). Therefore, by RHR #2, the direction of $I_1$ must be clockwise.

(ii) Designating upward as the positive vertical direction:

$$ |B_{1,\text{near,center}}| - |B_{1,\text{far,center}}| - |B_{2,\text{center}}| = 0 $$

Or:

$$ B_{1,\text{near,center}} - B_{1,\text{far,center}} = B_{2,\text{center}} $$

Or:

$$ \mu_0 I_1/\{2\pi(r_{1,near})\} - \mu_0 I_1/\{2\pi(r_{1,\text{far}})\} = \mu_0 I_2/(2R_2) $$

Or:

$$ \mu_0 I_1/\{2\pi(d_3 + d_2/2)\} - \mu_0 I_1/\{2\pi(d_1 + d_3 + d_2/2)\} = \mu_0 I_2/(2d_2/2) $$

Factoring out $I_1$:

$$ I_1\{\mu_0/\{2\pi(d_3 + d_2/2)\} - \mu_0/\{2\pi(d_1 + d_3 + d_2/2)\}\} = \mu_0 I_2/(2d_2/2) $$

Therefore:

$$ I_1 = \mu_0 I_2/(2d_2/2)/\{\mu_0/\{2\pi(d_3 + d_2/2)\} - \mu_0/\{2\pi(d_1 + d_3 + d_2/2)\}\} $$
5. f. A straight wire carries a steady current, $I_1 = 10.0 \text{ A}$, along the entire $z$-axis. The current is directed in the positive $z$-direction.

In the $x$-$y$ plane: Let $\angle 0^\circ$ (south) be the positive $x$-axis; and let $\angle 90^\circ$ (up) be the positive $y$-axis.

(i) Give at least three reasons why there would be zero magnetic force acting on a particle located at point A. (Use the above angle measurement system to specify all known angles in your reasoning.)

(ii) Suppose that an electron is at point A but moving directly toward the origin. 
   Evaluate (T/F/N) the following statement: The force on that electron is directed westward. 
   As always, you must explain your reasoning with any valid mix of words, equations, diagrams, etc.
5.  (iii) Suppose you want to create a net magnetic field at point A that is directed to the north. Through what point in the x-y plane would you need to run another long, straight current $I_2$ that is equal in magnitude to $I_1$ and either parallel or anti-parallel to $I_1$? Be sure to show all your work.

The sum of $B_1$ and $B_2$ must point directly north (to the left on the page). There are many possibilities; a few are shown here ($B_{2a}$, $B_{2b}$, $B_{2c}$, and $B_{2d}$), to illustrate.

Of course, the greater the magnitude of $B_2$, the closer that means you must position $I_2$ to point A, because $B_2 = \mu_0 I_2/(2\pi r)$, where $r$ is the distance from $I_2$ to point A.

One easy solution would be case $B_{2b}$, which is equal in magnitude to $B_1$ (thus they form an isosceles triangle, with $B_{net}$ as their vector sum).

And if $B_2 = B_1$, this implies that $I_2$ and $I_1$ would be equidistant from point A.

Assuming that case, here are two possibilities for the location of $I_2$ (depending on whether it is parallel or anti-parallel to $I_1$):

Clearly, A is at the vertex of an isosceles triangle for either case, so by simple geometry:

$I_{2\parallel}$ could be located at $(-6, 0, 0)$

$I_{2\text{anti-parallel}}$ could be located at $(0, 2, 0)$
6. a. Two identical copper pipes are partially buried, vertically upright in level ground (like fenceposts), 14.2 cm apart. Pipe 1 is directly north of pipe 2. An electric current of 980 A is flowing vertically up pipe 2. An unknown electric current is flowing in pipe 1. The magnetic field at point A, midway between the pipes, is 6.00 x 10^{-3} T, directed due west. Find the magnetic field strength and direction at point X, which is 6.20 cm north of pipe 1.

(Each current flows in a separate circuit that continues underground and from the top of the pipe, in a plane perpendicular to the page. Only the parts of each pipe shown here affect the points in question.)

Here’s a complete diagram of the problem, showing all six geographic directions:

![Diagram of the problem](image)

The total field at point A: \( B_{AT} = B_{A,1} + B_{A,2} \) (This is a vector sum.)

And:

\[
B_{A,2} = \frac{\mu_0 I_2}{2\pi r_2}
\]

(And by RHR-2, \( B_{A,2} \) is directed west ( )

\[
= (4\pi \times 10^{-7})(980)/[(2\pi)(.071)] = 2.76 \times 10^{-3} \text{T}
\]

Since \( B_{AT} \) is also directed west and is stronger than \( B_{A,2} \), this means \( B_{A,1} \) must also be directed west.

Therefore \( I_j \) flows downward by RHR-2).

Thus, for \( B_{A,1} \):

\[
B_{A,T} - B_{A,2} = B_{A,1} = \frac{\mu_0 I_j}{2\pi r_1}
\]

Solving for \( I_j \):

\[
I_j = \frac{2\pi r_1/\mu_0(B_{A,T} - B_{A,2})}{2\pi r_1/\mu_0 - r_1/d}
\]

\[
= \left[2\pi(0.071)/(4\pi \times 10^{-7})\right] (0.00600 - 0.00276) = 1150 \text{A}
\]

Now we know everything necessary to find the total B-field at point X:

\( B_{X,T} = B_{X,1} + B_{X,2} \) (This is a vector sum.)

These vectors are in opposite directions (by RHR-2), so, assigning west to be positive and east to be negative, we take the difference of their magnitudes:

\[
B_{X,T} = -\frac{\mu_0 I_j}{2\pi d} + (\mu_0 I_j)/[2\pi(r_1+r_2+d)]
\]

\[
= [\mu_0/(2\pi)][I_j/(r_1+r_2+d) - I_j/d]
\]

\[
= [(4\pi \times 10^{-7})/(2\pi)][980/.204 - 1150/.062] = -2.75 \times 10^{-3} \text{T}
\]

That’s 2.75 x 10^{-3} T east.
6.  b. Two parallel wires are oriented vertically (so their lower ends rest on the level earth; their upper ends point up toward the sky). The wires carry unknown electric currents, \( I_1 \) and \( I_2 \). The wires are positioned on a 3-D coordinate axis system (all coordinates in cm). The lower end of \( I_1 \) is located at the origin \((0,0,0)\); its upper end at \((0,0,43)\). The lower end of \( I_2 \) is located at \((2,0,0)\); its upper end at \((2,0,43)\). So each wire is 43 cm long. If an electron were moving directly northward at 58769 m/s as it passes through point \( P (1,1,20) \), the magnetic force exerted on it at \( P \) would be zero. However, if an electron were moving directly westward at 35604 m/s as it passes through point \( P \), the magnetic force exerted on it at \( P \) would instead be \( 9.70 \times 10^{-18} \) N, directed vertically upward (out of the earth). Find the magnitude and direction of the magnetic force exerted by wire 1 on wire 2.

At just 20 cm off the ground, point \( P \) is well within the fields \( B_1 \) and \( B_2 \) produced by the currents; those currents flow from 0 to 43 cm in altitude, so their fields would surround them throughout their heights.

If the force on a westbound electron at point \( P \) is upward, then by RHR #1, the total \( B \)-field it is encountering at point \( P \) must be directed due north. (This is also confirmed by the fact that a northbound electron feels no force from the field—it wouldn’t, since it’s traveling parallel to the field: \( \sin \theta = 0 \))

The total field, \( B_T \), is the vector sum of \( B_1 \) and \( B_2 \), and \( B_T \) is due north. Therefore: \[
B_{1x} + B_{2x} = 0.
\]

In other words, in terms of only their magnitudes:

\[
B_{1x} = B_{2x}
\]

Looking at the geometry of the situation and using RHR #2 to judge the possible directions of the fields:

Depending on the direction of \( I_1 \), the direction of \( B_1 \) at point \( P \) is either \( \angle 135^\circ \) or \( -45^\circ \). So in terms of magnitude, the two vector components form a 45° right triangle (equal legs):

\[
B_{1x} = B_{1y}
\]

Depending on the direction of \( I_2 \), the direction of \( B_2 \) at point \( P \) is either \( -135^\circ \) or \( 45^\circ \). So in terms of magnitude, the two vector components form a 45° right triangle (equal legs):

\[
B_{2x} = B_{2y}
\]

Thus, no matter which way the currents are flowing, \( B_{1y} = B_{1x} = B_{2x} = B_{2y} \)

Therefore, the magnitudes of the two fields must be equal:

\[
B_1 = B_2
\]

In other words: \[
\mu_0 I_1/(2\pi r_1) = \mu_0 I_2/(2\pi r_2)
\]

But from the geometry, we can see that \( r_1 = r_2 \). Therefore:

\[
I_1 = I_2
\]

Also, knowing that the total field at \( P \) must point due north, we can see that the correct directions of \( B_1 \) and \( B_2 \) must be \( 135^\circ \) and \( 45^\circ \), respectively. No other possible combination of directions for for those vectors would produce a vector sum directed to the north.

Therefore, by RHR #2: \( I_1 \) must be directed upward; \( I_2 \) must be directed downward.

Anti-parallel currents repel one another, so the force by wire 1 on wire 2 is to the right (east).

And the magnitude of that force is given by \( F_{12} = I_1 L_2 B_1 = I_1 L_2 [\mu_0 I_1/(2\pi r_1)] = I_1 L_2 [\mu_0 I_2/(0.04\pi)] \)

\( L_2 = 0.43 \) m, and \( r_{12} = 0.02 \) m. We need only find the magnitude of the matching currents, \( I_1 \) and \( I_2 \).

Start by calculating the total field magnitude, \( B_T \), at \( P \), using the known force on the westbound electron:

\[
F = qvB_T, \text{ therefore } B_T = F/(qv)
\]

But \( B_T \) is the sum of two equal components that are at right angles to one another: That is, \( B_T \) is the hypotenuse of a 45° right triangle, which is always \( \sqrt{2} \) times the length of either side. Thus, \[
B_{tp} = B_T/(\sqrt{2}) = F/((\sqrt{2})qv)
\]

But we also know that \[
\mu_0 I_p/(2\pi r_1 p) = F/((\sqrt{2})qv)
\]

Therefore:

\[
\mu_0 I_p/(2\pi (\sqrt{2})(0.01)) = F/((\sqrt{2})qv)
\]

Solve for \( I_p \): \[ I_1 = 0.02\pi F/(\mu_0 qv) = 0.02\pi(9.70 \times 10^{-18})/[(4\pi \times 10^{-7})(35604)(1.60 \times 10^{-19})] = 85.138 \text{ A}
\]

So \( F_{12} = I_1 L_2 [\mu_0 I_1/(0.04\pi)] = (85.138)(.43)(4\pi \times 10^{-7})(85.138)/(0.04\pi) = 0.0312 \text{ N to the right (east)} \)
6. c. A cyclotron that accelerates protons has an outer radius of 0.350 m. The protons are emitted nearly at rest from a source in the center (assume this source is near the side of the gap that is initially positive) between the two “dees.” Each time the protons move through the gap, they are accelerated through a potential difference of 600 V (made possible because the voltage source alternates; the gap sides reverse as to which is + vs. −). The dees’ semi-circular tops and bottoms face the poles of a magnet whose field is a constant 0.800 T.

(i) Find the cyclotron frequency for the protons in this cyclotron.

\[ r = \frac{mv}{(qB)} \quad \text{and} \quad v = \frac{2\pi r}{T} = 2\pi rf \]

Combine and solve for \( f \):

\[ f = \frac{qB}{(2\pi m)} \]

\[ = (1.60 \times 10^{-19})(0.800)/\left[2\pi(1.67 \times 10^{-27})\right] \]

\[ = 1.22 \times 10^7 \text{ Hz} \quad (12.2 \text{ MHz}) \]

(ii) Find the speed at which protons exit the cyclotron and their kinetic energy at that time.

\[ v_{exit} = 2\pi r_{exit} \cdot f = (2\pi)(0.350)(1.22 \times 10^7) = 2.68 \times 10^7 \text{ m/s} \]

(iii) How many revolutions does a proton make in the cyclotron?

In each revolution, a proton is accelerated through \( \Delta V_{rev} \) 1200 V (two passes across the gap) per revolution. Therefore the work done on it over \( n \) revolutions is given by \( W_{ext.rev} = nq\Delta V \).

The particles started from rest, so this work must equal the exiting kinetic energy: \( nq\Delta V = (1/2)m v_{exit}^2 \)

Therefore: \( n = \frac{mv_{exit}^2/(2q\Delta V_{rev})}{(1.67 \times 10^{-27})(2.68 \times 10^7)^2/[2(1.60 \times 10^{-19})(1200)]} \)

\[ = 3.13 \times 10^3 \text{ revolutions} \]

(iv) How much time does the proton spend in the cyclotron?

Total time \( t_{total} = nT \), where \( T \) is the period of one revolution. But \( T = 1/f \).

\[ t_{total} = n/f = (3.13 \times 10^3)/(1.22 \times 10^7) = 2.57 \times 10^{-3} \text{ s} \quad (2.57 \text{ ms}) \]

d. One kind of mass spectrometer selects ions having the same velocity, then sorts them according to mass, as follows (see diagram): The ions pass through a velocity selector, which is an electric field, \( E \), produced by oppositely charged plates, combined with a magnetic field, \( B \), that is perpendicular to that electric field and also to the ions’ path. Only the ions that pass through the crossed \( E \) and \( B \) fields without altering their path will enter into a region where a second magnetic field \( B' \) sends them into circular paths of various radii \( r_1, r_2, etc. \)

A photographic plate then indicates their arrival at various points. Show that for these ions, the ratio of charge to mass is given by:

\[ \frac{q}{m} = \frac{E}{rBB'} \]

If an ion travels at constant velocity through the perpendicular fields \( E \) and \( B \), then those two force magnitudes must be equal:

\[ F_E = F_{mag} \quad \text{That is} \quad qE = qvB \quad \text{Or} \quad v = E/B \]

Then the radius of the ion’s circular path in the second field \( B' \) is given by:

\[ r = \frac{mv}{(qB')} \]

That is: \( \frac{q}{m} = \frac{v}{(rB')} \) \quad \text{But:} \quad v = E/B \quad \text{Substituting:} \quad \frac{q}{m} = \frac{E}{(rBB')} \]
Refer to the overhead view shown here. A particle of mass $m$ is moving horizontally, directly north when it enters the shaded area shown here. Throughout the rectangular shaded region is a uniform vertical magnetic field of magnitude $B$. The borders of this shaded region are aligned with the four compass directions (north, south, east, west)

While in the magnetic field, the particle travels directly east along the dotted line segment shown, which is midway between the charged parallel plates shown and a distance $d$ from the south edge of the B-field. The plates each have a length $L$, and between them they produce a uniform electric field of magnitude $E$.

Sometime after exiting the electric field, the particle also exits the magnetic field. At all times, the particle is traveling horizontally. You may assume the following values to be known, so they may appear in your solutions/answers: $m, B, L, E, d$

(i) How far apart are the points where the particle enters and exits the magnetic field?

This particle is moving horizontally in a vertically directed magnetic field. So the B-field is perpendicular to the charge’s velocity at all times. A charged particle moving in a perpendicular magnetic field will (if acted upon by no other force) move in a circular path (of radius $r = mv/qB$).

So, except when the particle is moving between the plates (where it’s in both a magnetic and electric field), it must be moving in a circular path.

The only way it could do this with the information given (entering the field northward, but traveling between the plates eastward) is as shown here.

From this diagram it’s clear that the distance, $D$, between the entry and exit points is simply

$$D = L + 2d$$

(ii) How long (how much time) is the particle within the magnetic field?

Using the above reasoning, the total path length traveled is the length of the plates, $L$, plus two quarter-circles of radius $d$.

That is:

$$\Delta s = L + 2(\pi d/4)$$

$$= L + \pi d$$

The speed of the particle can be determined from the fact that it maintained a constant velocity while traveling between the plates. That is, the magnetic and electric forces must have been equal in magnitude (but opposite in direction):

So, for a constant $v$:

$$qE = qvB$$

Thus:

$$v = E/B$$

Then just kinematics:

$$v = \Delta s/\Delta t$$

Substitute:

$$E/B = (L + \pi d)/\Delta t$$

Solve for $\Delta t$:

$$\Delta t = B(L + \pi d)/E$$
6. e. (iii) There are at least two possible explanations for the particle’s motion, depending on the sign of its charge and the directions of the B-field and E-field. **Give both explanations** — and be specific: For each case, draw the path of the particle, give the sign of its charge, and indicate the directions of the E-field and the B-field.

Here is the diagram showing the field directions assuming that the particle was **positively charged**:

![Diagram](image1)

And here is the diagram showing the field directions assuming that the particle was **negatively charged**:

![Diagram](image2)
7. a. The diagram shows an overhead (“edge-on”) view of a loop of current-carrying wire that is entirely within a steady magnetic field. Evaluate each statement (T/F/N). You must fully and correctly explain your reasoning.
   (i) If this is an electric motor, the loop is rotating clockwise.

   (ii) If this is a generator, the loop is rotating counter-clockwise.

   (iii) The loop itself is a magnet whose north pole is facing this way:

b. Two solenoids (#1 and #2) are adjacent to each other, as shown. Evaluate each statement (T/F/N). As always you must fully and correctly explain your reasoning.
   (i) If #2 were connected to a light bulb, then #1 connected to a battery, the light could glow steadily.
      False. Induction works only during the time magnetic flux is changing.
   (ii) If #1 were connected to a light bulb, then #2 connected to a generator, the light could glow steadily.
      True. A generator can produce alternating current, which will produce an alternating induced voltage across (and thus an alternating current through) the light bulb, which (assuming the frequency is sufficiently rapid—for example, 60 Hz is typically plenty) to make it glow steadily.
   (iii) If you were to push a bar magnet, north end first, into the lower end of #1, both solenoids would become magnets (temporarily), with their north poles at their lower ends.
      Not enough information. By Lenz’ Law, increasing the flux into the lower end of #1 (that’s what the inserted north pole of the bar magnet does) will induce #1 to do the opposite—send its B-field out of that lower end. Thus, #1 has temporarily become a magnet with its north pole downward. As for #2, the upward movement of the magnet may in fact produce a gain in downward flux through its coils, which may or may not offset the appearance of some of #1’s induced field upward through its coils. More information is needed.

c. A steady magnetic field exists in the large shaded region. A proton moves at all times in the circular path shown (ignore any field it causes). Two plain metal loops are adjacent to one another, as shown (but they never touch). Evaluate each statement (T/F/N). As always, fully explain your reasoning.
   (i) Moving loop 2 to the right will induce a counter-clockwise current in it.

   (ii) Moving loop 2 to the right will induce a counter-clockwise current in loop 1.

   (iii) Moving loop 1 to the right will induce a clockwise current in loop 2.

d. Evaluate (T/F/N) the following statement. Justify your answer fully with any valid mix of words, drawings and calculations. In a uniform magnetic field provided by a bar magnet, if the coil of a generator starts with its normal perpendicular to the field but is then turned 90° so that its normal faces the bar magnet’s north pole, then, viewing the coil from the bar magnet’s north pole, the current induced in the coil during the turn will be clockwise.
   False. As it turns toward the north pole of the magnet, the coil is gaining flux in the direction away from that north pole. Therefore, by Lenz’s Law, the coil’s induced current will oppose that increase by trying to send flux the other way (back toward the magnet’s north pole). Viewing this induced response from that magnet’s north pole, by RHR for fields, that induced current would be counter-clockwise.
7. d. A rectangular copper loop lying in a perpendicular magnetic field doubles both its length and width. Evaluate each statement (T/F/N). As always you must fully and correctly explain your answer and reasoning.

(i) The magnetic flux is doubled.
   False. The area of the loop is quadrupled, so the flux is quadrupled.

(ii) The magnetic flux remains constant.
   False. The area of the loop is quadrupled, so the flux is quadrupled.

(iii) The induced voltage must be 4 volts.
   False. Must be? No. Could be? Yes. It depends on the strength of the magnetic field and the time interval over which the area of the loop is quadrupled.

e. Evaluate each statement (T/F/N). As always you must fully and correctly explain your answer and reasoning.

(i) Magnetic braking uses magnetic induction.
   True. The eddy currents that the magnetic field exerts the braking forces on are induced currents.

(ii) If you pull a loop lying horizontally on the ground directly away from a vertical current-carrying wire, no current will be induced in the loop.
   True. No flux goes through the loop in this orientation, no matter how near or far it is from the wire.

(iii) Whenever there is an magnetically induced voltage, there is also an induced current.
   False. There is always an induced voltage (an E-field) around the induction loop, but if there is no conductive path along that loop, there will be no current (recall the slotted “non-jumping” ring in the lab).

(iv) If a loop of wire lies at rest on a floor, with a simple bar magnet standing on its north pole at rest in the middle of that loop, current will flow counter-clockwise around the loop (as viewed from above).
   False. If everything’s at rest, there is no change in flux, therefore no induced field or current.

(v) If a loop of wire lies at rest on a floor, with a simple bar magnet standing on its north pole at rest in the middle of that loop, then if you lift the magnet straight upward, current will flow clockwise around the loop (as viewed from above).
   True. As the magnet lifts, less and less of its north-pole flux is downward through the loop, so its induced field would oppose that reduction—add downward flux back through it. The RHR for field will indicate that the induced current direction will be clockwise as viewed from above.

(vi) If you move a conductive plate (with its normal pointing upward) horizontally into a uniform magnetic field directed vertically downward, the resulting eddy currents will flow counter-clockwise (viewed from above).
   True. As the plate moves into the field, every conductive path that’s partially in field at any point will be gaining downward flux as a result of the plate’s motion. Therefore, the induced current around every such path will be counter-clockwise in order to oppose that downward flux increase (by sending flux lines upward, as the RHR for field will demonstrate).

(vii) Wb/s has units equivalent to W/A.
   True. Wb/s is the rate of flux change, which is equivalent to voltage. Watts/Amp is also voltage: \[ P = AV \]

(viii) A metal sheet moving horizontally (no twisting or turning) and entirely within a uniform vertical B-field, will experience no magnetic braking force.
   True. There’s no change in flux for the motion described.

(ix) If you connect a simple battery to a transformer, you can step its voltage either up or down by adjusting the number of loops in either the primary or secondary coil.
   False. You cannot induce a steady voltage via the steady current and field of a battery (a dc current loop); only a changing field (via a changing current—such as an alternating current—will do that).

(x) A transformer could work properly if connected to an electrical generator.
   True. A generator is a source of alternating current, and that continuing change of current provides a continuing change of flux through the secondary coil (the one not connected to the generator), which is required for proper operation (stepping up or stepping down the primary voltage).

(xi) Both of the coils of a properly operating transformer must be connected to an AC power source.
   False. Only the primary side is connected. The other side is not conductively connected to any power source. The power to it is transmitted via the (changing) magnetic field shared by the two coils.

(xii) If a circular wire loop is entirely within a uniform magnetic field, and you spin the loop along an axis (through its diameter) that is parallel to the field, no current is induced in the loop.
   True. There’s no change in flux for the motion described.
8. a. The wingspan (tip to tip) of a Boeing 747 jetliner is 59 m. The plane is flying horizontally toward magnetic south at a speed of 220 m/s. The vertical component of the earth’s magnetic field is $5.0 \times 10^{-6}$ T. The horizontal component is $11 \times 10^{-6}$ T. Find the induced emf between the wing tips.

Since the plane is flying directly toward a magnetic pole, the horizontal component of the earth’s field is parallel to the plane’s motion, so that component does not exert any forces on the conductive charges in the metal wings. Only the vertical component is relevant here. The motional emf induced by a conductive bar of length $L$ moving at speed $v$ through a perpendicular magnetic field $B$ is given by $E = vBL = (220)(5 \times 10^{-6})(59) = 64.9$ mV

b. Suppose you have a conductive loop and a magnetic field field positioned as shown here. There are about a dozen different “adjustments” you could make to this basic initial situation to cause electric current to flow in the loop. Two such adjustments, for example, would be to increase or decrease the magnetic field magnitude. List eight other adjustments that would also induce a current in the loop.

Here are ten: 
- Rotate the B-field clockwise (as viewed here); rotate the B-field counter-clockwise.
- Increase the area of the loop; decrease the area of the loop.
- Rotate the loop clockwise (as viewed here); rotate the loop counter-clockwise.
- Move the loop to the left (as viewed here); move the loop to the right.
- Move the loop upward (as viewed here); move the loop downward.

(c. A conducting loop of wire is placed in a magnetic field that is normal to the plane of the loop. Which of the following actions will not result in an induced current in the loop?

(i) Rotate the loop about an axis that is parallel to the field and passes through the center of the loop.
   This is the only case that will not change the flux through the loop, so this is the only case that will not induce a current through the loop.

(ii) Increase the strength of the magnetic field.

(iii) Decrease the area of the loop.

(iv) Decrease the strength of the magnetic field.

(v) Rotate the loop about an axis that is perpendicular to the field and passes through the center of the loop.

d. Find the direction of the induced current in resistor $R_A$ while...

(i) coil B is moved toward coil A.
   **Left to right** (opposite the current from the battery in B).
   If the current in B is traveling clockwise as you look into its left end, its right end will be its north pole, sending flux into the left end of A, which will respond by trying to send flux out of its left end, thus requiring a current flowing oppositely to B.

(ii) while the resistance $R_B$ is increased.
   **Right to left** (same direction as the current from the battery in B).
   A larger resistance will decrease the current in B, therefore decrease the flux out of B’s right end into A’s left end, so A will respond oppositely, requiring a current flowing right to left.

(iii) while coil B is rotated around a vertical axis through its center (during the first 90° of rotation).
   **Right to left** (same direction as the current from the battery in B).
   Rotating B will decrease the flux that enters into into A’s left end, so A will respond oppositely, requiring a current flowing right to left.

e. As you insert the north pole of a bar magnet into one end of a solenoid, if there is a compass sitting just outside the other end of the solenoid, how will that compass’s needle react? (Assume that the compass reacts only to the solenoid and that the solenoid’s two ends are connected electrically so that they form a closed conductive path.)

The solenoid will send flux back against the north end of the bar magnet; that end of the solenoid becomes its north pole, and the other end is its south pole. Thus the compass at its other end will point **toward** that end.
8. f. The drawing shows that a uniform magnetic field is directed perpendicularly into the plane of the paper and fills the entire region to the left of the y axis. There is no magnetic field to the right of the y axis. A rigid right triangle ABC is made of copper wire. The triangle rotates counterclockwise about the origin at a point C. What is the direction (cw or ccw) of the induced current when the triangle is crossing

(i) the +y axis?

(ii) the –x axis?

(iii) the –y axis?

(iv) the +x axis?

9. a. (i) An electric generator is a device which converts mechanical energy into electrical energy.

(ii) As you push a conductive loop into a magnetic field, energy is produced (as current flows through a resistor). Where does this energy come from?

(iii) Express magnetic flux in terms of only basic SI units (kg, m, s and C). Simplify as much as possible.

By Faraday’s Law, flux change has units of V·s (J·s/C). Thus: \( N \cdot m \cdot s / C = \text{kg} \cdot m^2 / (C \cdot s) \)

(iv) Express the rate of change of magnetic flux in fundamental (“base”) SI units (kg, m, s and C).

Simplify/reduce this as much as possible.

By Faraday’s Law, flux change per unit time has units of voltage (J/C). Thus: \( N \cdot m / C = \text{kg} \cdot m^2 / (C \cdot s^2) \)

b. A sheet of aluminum, with its normal oriented due west, falls vertically toward the earth’s surface. As it falls, it encounters a uniform magnetic field that is oriented due west.

(i) Viewed from the east (i.e. looking west), what direction do the eddy currents flow in the sheet as it enters the field?

(ii) Viewed from the east (i.e. looking west), what direction do the eddy currents flow in the sheet as it leaves the field?

(iii) The current dissipates electrical energy. In what form is that energy before the sheet has entered the field?

(iv) In what form is that energy after the current has ceased to flow?

c. Lying on level ground in a B-field initially directed vertically upward, a copper loop stretches, doubling its area. Meanwhile, the field doubles in strength but shifts to an angle of 15° above the horizontal. By what percentage has the magnetic flux through the loop changed?

\[
\Delta\% \Phi = 100(\Phi_f - \Phi_i)/\Phi_i = 100(\Phi_f/\Phi_i - 1)
\]

Initial: \( B_i \), \( A_i \), \( \phi_i = 0^\circ \)

Final: \( B_f = 2B_i \), \( A_f = 2A_i \), \( \phi_f = 75^\circ \)

\( \Phi_f = B_f A_f \cos \phi_f = 2B_i (2A_i) \cos 75^\circ = (1.0353)B_i A_i \)

Thus:

\( \Delta\% \Phi = 100(\Phi_f/\Phi_i - 1) = 100(1.0353 - 1) = 3.53\% \)

d. A 14-turns square conductive loop has dimensions 2.00 cm by 3.00 cm. Directed at an angle of 25° from the plane of the loop, there is a time-dependent magnetic field, \( B(t) = (0.20t^2 - 0.40t^3 + 0.800) \). What will be the magnitude of the induced emf at \( t = 2s \)?
9. e. A circular coil of wire has 25 turns and has a radius of 0.75 m. The coil is located in a variable magnetic field whose behavior is shown on the graph here. At all times, the magnetic field is directed at an angle of 75° relative to the plane of the loops. What is the magnitude of the average emf induced in the coil in the time interval from \( t = 5.00 \) s to 7.50 s?

\[ |\Delta V| = \frac{d\Phi}{dt} = d\left(NA\cos\phi\right)/dt = (NA\cos\phi)dB/dt, \]

since \( N, A \) and \( \phi \) are all constant.

\[ \frac{dB}{dt} = \frac{(0.80 - 0.40)}{2.50} = 0.16 \ T/s \]

Therefore:

\[ |\Delta V| = (25)(\pi)(0.75)^2(\cos15°)(0.16) = 6.83 \ V \]

f. Magnetic resonance imaging (MRI) is a medical technique for producing pictures of soft body tissue. In this technique, the patient is placed for a few minutes within a very strong magnetic field (1.75 T). Among other safety concerns is the issue of sudden power loss to the imaging equipment, which would result in a sudden disappearance of the magnetic field. Suppose the average surface area of the body exposed to magnetic flux is about 0.0362 \( \text{m}^2 \). Then, assuming that the maximum allowable induced voltage within the body is 0.01 V, what minimum time must elapse while the magnetic field is dropping to zero?

The maximum allowable induced voltage magnitude is given by Faraday’s Law:

\[ |V_{\text{max}}| = \frac{\Delta \Phi}{\Delta t}, \]

where \( \Phi = BAC\cos\phi \) and \( \Delta \Phi = B FA\cos\phi_i - B FA\cos\phi_f \). Solve this for \( \Delta t \):

\[ \Delta t = \frac{(B FA\cos\phi_i - B FA\cos\phi_f)}{|V_{\text{max}}|} \]

In this case, neither \( A \) (the area of the body’s largest conductive path) nor \( \phi \) (the angle between the normal to any such path and the B-field—in this case, 0°) will be changing; only \( B \) (decreasing from 1.75 T to 0 T).

So the equation simplifies:

\[ \Delta t = \frac{A_f (B_f - B_i)}{V_{\text{max}}} \]

The numbers:

\[ \Delta t = \frac{(0.0362)(0 - 1.75)}{0.010} = 6.34 \text{ s} \]

To limit the induced voltage magnitude to the specified \( V_{\text{max}} \), the field must be turned off over a time interval of at least 6.34 seconds.

g. A rectangular copper loop is moved horizontally (i.e., parallel to the level ground) along the east-west axis only—without twisting or lifting. During its motion, it is at least partially within a region containing a vertical uniform magnetic field.

The resulting current-time graph is shown here.

Evaluate (T/F/N) each statement.

As always, you must justify your answers fully with any mix of valid explanations, drawings and calculations.

(i) In the time interval \( 5s < t < 7s \), the loop was completely within the field.

(ii) In the time interval \( 2s < t < 9s \), the net force exerted by the field on the loop was always in the same direction.

(iii) The loop was moving faster in the time interval \( 7s < t < 9s \) than in the time interval \( 2s < t < 5s \).
10. a. A uniform horizontal magnetic field, directed due north, exists all over a certain area of the earth (assume this area is basically flat). A circular loop of wire (diameter = 1.40 m) is rotating around an axis through its diameter at a steady rate of 1200 rpm. That axis of rotation is also horizontal, but it is directed at an angle of 30° west of north. The resistance of the wire loop is 100 \( \Omega \), and the field strength is 0.750 T. What average power is generated as the loop rotates through 1/4 of a revolution, starting with its normal directed vertically upward?

b. A 20-cm-diameter loop of wire has a resistance of 62 \( \Omega \). It is initially in a uniform 0.87-T magnetic field with its normal at an angle of 39° with respect to the field. In a time interval of 0.15 s, the loop is turned until its normal aligns with the B-field. Then, in an additional time interval of 0.14 s, the loop is completely withdrawn (without turning) from the field. [Again, using simple averaging, find the total electrical energy dissipated during this entire two-step process.

The two steps—two instances of induced currents (and therefore dissipations of power)—occur at entirely separate times—treat them independently. In each case, the induced voltage will produce a power of \( P = \Delta V^2 / R \) in the resistor. Thus the energy dissipated in the time \( \Delta t \) during which the voltage exists is \( E = P\Delta t = \Delta V^2 \Delta t / R \).

The magnitude of \( V \) is given by Faraday’s Law: \( \Delta V = (B_f A_f \cos \phi_f - B_i A_i \cos \phi_i) / \Delta t \)

Substituting, the energy dissipated is thus: \( E = [(B_f A_f \cos \phi_f - B_i A_i \cos \phi_i)^2 \Delta t / R] \)

Now just plug in the numbers that apply in each of the two situations:

**Step 1:** \( E_1 = [(0.87)(0.1^2 \pi)(\cos 0^\circ) - (0.87)(0.1^2 \pi)(\cos 39^\circ)]^2 / [(62)(0.15)] = 3.9893 \times 10^{-6} \) J

**Step 2:** \( E_2 = [(0.87)(0)(\cos 0^\circ) - (0.87)(0.1^2 \pi)(\cos 0^\circ)]^2 / [(62)(0.14)] = 8.6063 \times 10^{-5} \) J

**Total:** \( E_T = E_1 + E_2 = 9.01 \times 10^{-5} \) J

The total energy dissipated over both these steps is 9.01 \( \times 10^{-5} \) J.
10. c. The diagram shows the initial situation: There is a uniform magnetic field oriented into the page in the region shown. A single rectangular loop of wire, with initial position as shown, is partially in that field. Over a time interval of 0.375 seconds, the field strength, \( B \), is reduced from 1.20 T to 0.960 T.

With what average speed and in what direction would you need to move the loop during that time interval so that no net current would flow in the loop?

If there is to be no induced current, there must be no induced voltage and therefore no flux change over this interval: 
\[
V = \frac{\Delta \Phi}{\Delta t} = 0
\]
Therefore: 
\[
\Delta \Phi = B_f A_f \cos \phi_f - B_i A_i \cos \phi_i = 0
\]

Clearly, if you do nothing, a voltage and current will be induced because the decreasing field will reduce the number of field lines. So you must push the loop more into the field, so that a larger area can exactly counteract that loss of field lines. But how far must that be in the time allotted?

\[
B_f A_f \cos \phi_f = B_f A_i \cos \phi_i
\]

You’re not twisting the loop out of its current plane, so \( \cos \phi_f = \cos \phi_i \)

So: 
\[
B_f A_f = B_f A_i
\]

\( A_i \) is the initial amount of loop area encompassing field lines, given by \( A_i = (.840) \text{W} \). Likewise, the final area is \( A_f = (.840 + x) \text{W} \)

So solve this for \( x \):
\[
B_f(.840 + x) \text{W} = B_i(.840) \text{W}
\]

\[
x = B_f(.840)/B_f - 0.840
\]

Then \( v = x/\Delta t \):
\[
v = [(1.20)(.840)/.960 - .840]/.375 = 0.560 \text{ m/s}
\]

d. A rectangular loop of wire (dimensions 10 m x 4 m) slides horizontally eastward on a frictionless surface at a constant velocity of 2.00 m/s, first entering entirely, then exiting entirely, the vertical uniform magnetic field shown (field dimensions 14 m x 27 m). The resistance of the wire loop is 100 \( \Omega \), and the field strength is 2.00 T. At all times, including in both the “Begin” and “End” pictures below (viewing the situation from above), the loop is moving at 2.00 m/s. You ensure this steady speed throughout its trip by (sometimes) pushing on the it.

From the “Begin” picture to the “End” picture, what total work must you do on the loop?
10. e. In the drawing shown here, there is a circular loop (radius = 0.657 m) of copper wire that isn’t quite a complete circle (notice the gap). This copper has negligible value as a resistor. However, a 0.24-Ω resistor is connected to it at point A, as shown; the other end of that resistor is located at a pivot (point B) at the center of the circle.

Also connected to point B is a conductive bar that is free to slide on the copper, like a rail.

Suppose there is a uniform, external magnetic field \( B = 83.0 \text{ mT} \) directed out of the page throughout the entire drawing. If the conductive bar is sweeping around on the loop (in a counter-clockwise direction) at a constant angular speed of 5.0 rad/s as shown, find the direction of the current flowing in the resistor and the power it is producing.
11. a. You have a bar magnet (mass = 1.36 g) and a simple loop of metal wire (resistance = 0.402 $\Omega$).

**Trial 1:** You hold the loop motionless and drop the magnet from above it (with the magnet beginning at rest), letting it fall entirely through the loop (north end down) and then hit the floor with an impact speed of 5.79 m/s.

**Trial 2:** You cut the loop (i.e. make a thin gap in it—no loss of mass or shape), then repeat the above procedure (same starting height and orientation of the magnet, etc.). Now the impact speed is 5.86 m/s.

If the entire trip of the magnet during Trial 1 happened in 0.621 s, what effective current magnitude (i.e. just calculate it as a single, steady magnitude) must have flowed in the loop throughout that time?

b. A car accelerates uniformly from rest when a traffic light turns green. The permanent magnets in the car’s alternator start from rest and subsequently accelerate uniformly in the rotational sense. The magnitude of the magnets’ angular acceleration is $\alpha$; they produce a constant magnetic field of magnitude $B$; and they are surrounded by $N$ circular loops of wire of radius $r$. Use Faraday’s law to derive an expression for the magnitude of the induced emf in the wire in terms of $\alpha$, $B$, $N$, $r$ and $t$ (time).

\[ |\mathcal{E}| = |d\Phi_M/dt| = |d(BA\cos\phi)/dt| = BNA|d(\cos\phi)/dt| \quad B \text{ and } NA (= N\pi r^2, \text{ the area of the } N \text{ loops) are constant.} \]

Thus: \[ |\mathcal{E}| = BN\pi r^2 |\sin\phi||d\phi/dt| \]

**Rotational kinematics** for the magnet, assuming that $\phi_0 = 0$ at $t = 0$, so that $\Delta\phi = \phi_f = \phi(t)$ after $\Delta t = t_f = t$:

\[ \phi(t) = (1/2)\alpha t^2 \quad \text{and:} \quad |d\phi/dt| = \omega = \alpha t \]

Thus: \[ |\mathcal{E}| = BN\pi r^2 \alpha t |\sin[(1/2)\alpha t^2]| \]
A rectangular loop of wire of length \(L\), width \(W\), and resistance \(R\), is lying flat on level ground with an (artificially produced) external magnetic field, \(B_{\text{ext}}\), present inside the loop. This field is initially oriented at an angle of \(\theta\) off the vertical, as shown. Then, during a time interval \(\Delta t\), the field strength is steadily tripled and its orientation becomes vertical (downward). Point P is located on the ground a distance \(d\) outside the loop, as shown. What magnetic force (magnitude and direction) would be exerted on a proton traveling at speed \(v\) horizontally directly away from the loop through point P, at some moment during \(\Delta t\)?

**The list of known values:** \(L, W, R, B_{\text{ext}}, \theta, \Delta t, d, v, e, \mu_0\)

I. Find the magnetic flux in the loop at the start of \(\Delta t\) (using a downward normal for the loop):

Use: \(\Phi_i = B_{\text{ext}}(L \cdot W)\cos \theta\)  
Solve for: \(\Phi_i\)

Where: \(B_{\text{ext}}, L, W, \theta\) are given

II. Find the magnetic flux in the loop at the end of \(\Delta t\) (using a downward normal for the loop):

Use: \(\Phi_f = 3B_{\text{ext}}(L \cdot W)\cos 0^\circ\)  
Solve for: \(\Phi_f\)

Where: \(B_{\text{ext}}\) is given  
\(L\) is given  
\(W\) is given

III. Find the magnitude of the voltage induced around the loop during the time interval \(\Delta t\):

Use: \(|\Delta V| = \frac{|(\Phi_f - \Phi_i)|}{\Delta t}\)  
Solve for: \(|\Delta V|\)

Where: \(\Phi_f\) is from part II  
\(\Phi_i\) is from part I  
\(\Delta t\) is given

IV. Find the magnitude of the current induced around the loop during the time interval \(\Delta t\):

Use: \(|I| = \frac{|\Delta V|}{R}\)  
Solve for: \(|I|\)

Where: \(|\Delta V|\) is from part III  
\(R\) is given

V. By Lenz’s Law, the induced current, \(I\), is directed counter-clockwise (as viewed from above).

VI. Find the field strength due to the current loop \(I\) at point P. Only the near and far segments of the loop (i.e. the segments running along the width of the loop) will cause fields at point P. Using RHR #2 and defining upward (out of the earth) as the positive vertical direction…

Use: \(|B_{\text{P.I.total}}| = -B_{\text{P.I.near}} + B_{\text{P.I.far}} = -\mu_0|I|/(2\pi d) + \mu_0|I|/[(2\pi)(d+L)]\)  
Solve for: \(|B_{\text{P.I.total}}|\)

Where: \(\mu_0\) is known  
\(|I|\) is from part V  
\(d\) is given  
\(L\) is given

VII. At point P, the field strength of the current loop’s nearer segment is greater than that of its farther segment, so the direction of \(B_{\text{P.I.total}}\), the net field at point P, will be downward (into the earth).

VIII. Find the magnitude of the force that would be exerted on a proton moving through P:

Use: \(|F_{\text{mag}}| = |q|v|B_{\text{P.I.total}}|\sin \theta\)  
Solve for: \(|F_{\text{mag}}|\)

Where: \(|q| = e\) (known)  
\(v\) is given  
\(|B_{\text{P.I.total}}|\) is from part VI  
\(\theta = 90^\circ\) (because \(v\) is to right while \(B_{\text{P.I.total}}\) is downward)

IX. By the Right-Hand Rule for forces, the direction of \(F_{\text{mag}}\) would be into the page (X).
11. d. This problem deals with a steady uniform magnetic field \( B = 0.250 \, \text{T} \), directed into the page and located in the shaded region shown. Except for the field itself, each part (a or b) excludes all of the objects and data that are given in the other part.

(i) Suppose you have a rectangular copper loop with dimensions of 1.50 m x 2.50 m. Find the electrical resistance of this loop if you must exert a steady force \( F = 10 \, \text{N} \) on it in order to keep it moving at a steady speed \( v = 3 \, \text{m/s} \) to the right as it enters the field. At all times, the loop’s normal is aligned with the field.

![Diagram of a loop and a magnetic field B]

Faraday’s Law:

\[
|\Delta V| = \frac{d\Phi_f}{dt} = (d/dt)[B|A_{\text{flux}}|\cos\phi] = |B|\cos\phi\frac{d|A_{\text{flux}}|}{dt}
\]

We know:

\[ A_{\text{flux}} = l_{\text{flux}}w_{\text{flux}} \]

So:

\[ \frac{d|A_{\text{flux}}|}{dt} = (dl_{\text{flux}}/dt)(w_{\text{flux}}) + (dw_{\text{flux}}/dt)(l_{\text{flux}}) \]

But:

\[ dw_{\text{flux}}/dt = 0 \quad (w_{\text{flux}} = W_{\text{loop}}; \text{the width of the area gathering flux is not changing}) \]

Thus:

\[ |\Delta V| = |B|\cos\phi W_{\text{loop}} (dl_{\text{flux}}/dt) \]

Now, note:

\[ dl_{\text{flux}}/dt = v \]

So:

\[ |\Delta V| = BW_{\text{loop}}v \quad (= 0.250\cdot1.50\cdot3 = 1.125 \, \text{V}) \]

And:

\[ I = |\Delta V|/R = BW_{\text{loop}}v/R \]

The force (on the leading edge of the loop) that you must oppose to maintain speed \( v \) is:

\[ F_{\text{mag}} = ILB\sin\theta \]

Where:

\[ L = W_{\text{loop}} \]

And:

\[ \sin\theta = 1 \quad \text{(the angle \( \theta \) between \( B \) and \( I \) is 90°)} \]

Substituting:

\[ F_{\text{mag}} = (BW_{\text{loop}}v/R)(W_{\text{loop}})B = (B^2W_{\text{loop}}^2)(v/R) \]

Solve for \( R \):

\[ R = (B^2W_{\text{loop}}^2)(v/F_{\text{mag}}) = (0.250)^2(1.50)^2(3)/10 = 0.0422 \, \Omega \]

**Or, note this completely equivalent solution:** The mechanical power of your push provides the electrical power produced in the resistance.

That is:

\[ F \cdot v = \Delta V^2/R \]

So:

\[ R = \Delta V^2/(F \cdot v) = 1.125^2/(10 \cdot 3) = 0.0422 \, \Omega \]
A circular conductive loop of initial radius 1.00 m and resistance $R = 5 \ \Omega$ is lying at rest in the field, with its normal aligned with the field. At time $t = 0$, the radius of this loop begins to increase steadily: $r(t) = 1 + 3t \ (r \text{ measured in m, } t \text{ is in s}).$

Assuming no other changes to the loop, find the total magnetic field strength and direction at the center of the loop at time $t = 2.00 \text{ s}$.

(The drawing is not to scale. You may assume that the growing loop remains entirely within the field for the time interval $0 \leq t \leq 2.00 \text{ s}$)

Faraday’s Law: 

$$|\Delta V| = \frac{d\Phi_M}{dt}$$

$$= (d/dt)[|B||A_{\text{flux}}|\cos \phi]$$

$$= |B|(d|A_{\text{flux}}|/dt) \quad (\cos \phi = 1; \text{ the loop’s normal always aligns with } B)$$

We know: 

$$A_{\text{flux}} = \pi r_{\text{loop}}^2$$

So: 

$$d|A_{\text{flux}}|/dt = 2\pi r (dr/dt)$$

But: 

$$dr/dt = 3$$

Substituting: 

$$|\Delta V| = 6\pi r B$$

And: 

$$I = |\Delta V|/R$$

$$= 6\pi r B/R$$

The induced field magnitude at the center of the loop is:

$$B_{\text{induced}} = \mu_0 I/(2r)$$

$$= [\mu_0/(2r)]6\pi r B/R = 3\pi \mu_0 B/R$$

This induced field ($B_{\text{induced}}$) will \textbf{oppose} the external field ($B$), because the flux from $B$ is \textbf{increasing} as the loop grows in area. So the net field at the center will be the \textbf{difference} between the external field and the induced field at that point. Assigning now a positive direction into the page...

We have: 

$$B_{\text{NET}} = B - B_{\text{induced}}$$

$$= B - 3\pi \mu_0 B/R \quad (\text{into the page})$$
12. a. A rigid, conductive loop (with resistance $R$ and with normal oriented vertically) moves due eastward at constant speed $v$ for a time interval $\Delta t$. The loop’s enclosed area includes parts of three magnetic fields, each located and oriented as shown. Each field is independent, external, steady, motionless, and uniform—and each is limited to the region and dimensions indicated. Find the net force (magnitude and direction) exerted by the three fields on the loop during the time interval $\Delta t$. (Note: You may assume that the loop’s west end does not enter $B_1$, nor does its east end exit $B_3$.)

The list of known values: $R, v, \Delta t, L_{\text{loop}}, W_{\text{loop}}, B_1, L_1, W_1, B_2, L_2, W_2, B_3, L_3, W_3, \mu_0$.

I. The flux through the loop changes only because it moves farther into field $B_3$. There is no flux change due to field $B_1$ or field $B_2$, because the loop does not move either into or out of either of those fields.

II. Find the distance $\Delta x$ moved by the loop in the time interval $\Delta t$:

Use: $v = \Delta x / \Delta t$

Where: $v$ is given
$\Delta t$ is given

Solve for: $\Delta x$

III. Find the change in area capturing $B_3$ flux lines during the time interval $\Delta t$:

Use: $\Delta A = A_f - A_i = (L_{\text{capture},f} - L_{\text{capture},i}) W_{\text{loop}}$

Where: $L_{\text{capture},f} - L_{\text{capture},i} = \Delta x$ (from part II)
$W_{\text{loop}}$ is given

Solve for: $\Delta A$

IV. Find the flux change in the loop during $\Delta t$:

Use: $\Delta \Phi = B_3 \cos(\phi) \Delta A$

Where: $B_3$ is given
$\phi = 0^\circ$ (loop’s normal is into the page; aligns with $B_3$)
$\Delta A$ is from part II

Solve for: $\Delta \Phi$

V. Find the voltage magnitude induced in the loop during $\Delta t$:

Use: $|\Delta V| = \Delta \Phi / \Delta t$

Where: $\Delta \Phi$ is from part IV
$\Delta t$ is given

Solve for: $\Delta V$

VI. Find the current magnitude induced in the loop during $\Delta t$:

Use: $\Delta V = I_{\text{loop}} R$

Where: $\Delta V$ is from part V
$R$ is given

Solve for: $I_{\text{loop}}$

VII. Apply Lenz’s Law to find the direction of the current induced in the loop during $\Delta t$:

Since the loop is capturing more and more flux lines that are directed into the page, the induced current would flow so that its field would oppose that change—sending flux lines out of the page. Thus the induced current would flow counter-clockwise around the loop.
VIII. Find the magnitude of the force $F_1$ on the part of the loop flowing in the field $B_1$:

**Use:**
$$ F_1 = I_{\text{loop}} L_{\text{loop},1} B_1 \sin \theta_1 $$

**Solve for:** $F_1$

**Where:**
- $I_{\text{loop}}$ is from part VI
- $L_{\text{loop},1} = L_1$ (which is given)
- $\theta_1 = 90^\circ$ ($I_{\text{loop}}$ is to the left; $B_1$ is out of the page)

IX. Apply RHR #1 to find the direction of the force on the part of the loop flowing in the field $B_1$:

The current is to the left ($-x$-direction), and $B_1$ is out of the page ($+z$-direction), so the force $F_1$ is **up the page** ($+y$-direction).

X. Find the magnitude of the force $F_2$ on the part of the loop flowing in the field $B_2$:

**Use:**
$$ F_2 = I_{\text{loop}} L_{\text{loop},2} B_2 \sin \theta_2 $$

**Solve for:** $F_2$

**Where:**
- $I_{\text{loop}}$ is from part VI
- $L_{\text{loop},2} = L_2$ (which is given)
- $\theta_2 = 90^\circ$ ($I_{\text{loop}}$ is to the right; $B_2$ is into the page)

XI. Apply RHR #1 to find the direction of the force on the part of the loop flowing in the field $B_2$:

The current is to the right ($+x$-direction), and $B_2$ is into the page ($-z$-direction), so the force $F_2$ is **up the page** ($+y$-direction).

XII. Find the magnitude of the force $F_3$ on the part of the loop flowing in the field $B_3$:

**Use:**
$$ F_3 = I_{\text{loop}} L_{\text{loop},3} B_3 \sin \theta_3 $$

**Solve for:** $F_3$

**Where:**
- $I_{\text{loop}}$ is from part VI
- $L_{\text{loop},3} = W_{\text{loop}}$ (which is given)
- $\theta_3 = 90^\circ$ ($I_{\text{loop}}$ is up the page; $B_3$ is into the page)

XIII. Apply RHR #1 to find the direction of the force on the part of the loop flowing in the field $B_3$:

The current is up the page ($+y$-direction), and $B_3$ is into the page ($-z$-direction), so the force $F_3$ is **to the left** ($-x$-direction).

XIV. Find the total $x$-force on the loop:

**Use:**
$$ F_{x,T} = -F_3 $$

**Solve for:** $F_{x,T}$

**Where:**
- $F_3$ is from part XII

XV. Find the total $y$-force on the loop:

**Use:**
$$ F_{y,T} = F_1 + F_2 $$

**Solve for:** $F_{y,T}$

**Where:**
- $F_1$ is from part VIII
- $F_2$ is from part X

XVI. Find the magnitude of the total force on the loop:

**Use:**
$$ F_T = \left[ F_{x,T}^2 + F_{y,T}^2 \right]^{1/2} $$

**Solve for:** $F_T$

**Where:**
- $F_{x,T}$ is from part XIV
- $F_{y,T}$ is from part XV

XVII. Find the direction of the total force on the loop:

**Use:**
$$ \theta_T = \tan^{-1} \left[ \frac{F_{y,T}}{F_{x,T}} \right] $$

**Solve for:** $\theta_T$

**Where:**
- $F_{y,T}$ is from part XV
- $F_{x,T}$ is from part XIV

(90° < $\theta_T$ < 180°)
12. b. A charged particle (of mass \( m \) and charge \( \pm q \)), initially traveling east at speed \( v \), enters a region where there is a steady, vertical, uniform magnetic field, \( B_1 \), of unknown strength (and unknown direction—either up or down). The particle then exits the field to the north, as shown.

A long distance away lies a rectangular copper loop. It has a length \( L \), width \( W \), and total resistance \( R \). A distance \( d \) from the loop, a straight wire carries current \( I_3 \), as shown below. (The loop and the straight wire are so far from the particle and uniform field that neither situation affects the other.)

By coincidence, during the time interval when the particle is within \( B_1 \), another (external) vertical magnetic field appears and increases steadily inside the loop, achieving a final strength \( B_2 \) that is twice that of \( B_1 \) but oppositely directed. What average force (magnitude and direction) is exerted by the straight wire on the loop during this time interval?

Assume these are known values: \( m, q, v, x, L, W, d, R, I_3 \)

I. While it’s moving in the B-field, the particle’s path is a circle of radius \( x \):

\[
x = \frac{mv}{qB_1}
\]

Where:
- \( x \) is given
- \( m \) is given
- \( v \) is given
- \( q \) is given

II. \( B_1 \) is into the page (i.e. downward into the earth). We know this by RHR-1, since the positively charged particle is turned to its left.

III. Find the period, \( T \), of the particle’s motion:

\[
v = \frac{(2\pi x)}{T}
\]

Where:
- \( v \) is given
- \( x \) is given

IV. The particle is in the field for only 1/4 of a revolution, thus 1/4 of a period:

\[
\Delta t = \frac{T}{4}
\]

Where:
- \( T \) is from part III

V. Over the time interval \( \Delta t \), \( B_2 \)—directed oppositely to \( B_1 \) but twice the strength—arises inside the loop:

\[
B_2 = 2B_1
\]

Where:
- \( B_1 \) is from part I

(directed upward out of the page)

VI. The area of the loop that “captures” B-field lines is \( A = W \cdot L \), and this does not change:

\[
A_f = A_i = W \cdot L
\]

Where:
- \( W \) is given
- \( L \) is given

VII. The loop’s normal aligns with \( B_2 \) always:

\[
\phi_i = \phi_f = 0
\]

Solve for: \( B_1 \)

Solve for: \( T \)

Solve for: \( \Delta t \)

Solve for: \( B_2 \)

Solve for: \( A_f \)

Solve for: \( \phi_f \)
VIII. Find the voltage induced in the loop:

\[\Delta V_2 = \frac{\Delta \Phi}{\Delta t} = \frac{(B_f A_f \cos \phi_f - B_i A_i \cos \phi_i)}{\Delta t}\]

**Solve for:** \(\Delta V_2\)

**Use:**
- \(B_f = B_2\) (from part V)
- \(A_f\) is from part VI
- \(\phi_f = 0\) (from part VII)
- \(B_i = 0\) (given)
- \(A_i\) is from part VI
- \(\phi_i = 0\) (from part VII)
- \(\Delta t\) is from part IV

**Where:**

IX. Find the current induced in the loop:

\[I_2 = \frac{\Delta V_2}{R}\]

**Solve for:** \(I_2\)

**Use:**
- \(\Delta V_2\) is from part VIII
- \(R\) is given

X. The direction of \(I_2\) is **clockwise**, to oppose the rising flux of \(B_2\).

XI. The net \(x\)-force by the wire on the loop is **zero**. Every point on the left end of the loop is pushed to the right (by RHR #1); but every corresponding point on the right end of the loop is pushed with equal force magnitude to the left.

XII. Find the net \(y\)-force by the wire on the loop (using north as the positive direction along that axis). This is the vector difference between the repulsion force on the near current segment and the attraction force on the far current segment:

\[F_{net} = I_2 L \left( \frac{\mu_0 I_3}{2\pi d} \right) - I_3 L \left( \frac{\mu_0 I_2}{2\pi (d+W)} \right)\]

**Solve for:** \(F_{net}\)

**Use:**
- \(I_2\) is from part IX
- \(L\) is given
- \(I_3\) is given
- \(d\) is given
- \(W\) is given

XIII. The \(y\)-force on the nearer segment is stronger (its denominator is smaller—less distance); the total net force is the net \(y\)-force, which is **positive**—to the **north**.
12. c. A rectangular loop of wire, of length \( L \), width \( W \), and total resistance, \( R \), is lying flat on level ground at the location on the earth known as “Magnetic North.” (In this region, you may assume that the earth’s own magnetic field, \( B_{\text{earth}} \), is oriented entirely vertically.) Initially, the area inside the loop is also experiencing an (artificially produced) external field, \( B_{\text{ext}} \), that is oriented at an angle of \( \theta \) below the horizontal, as shown. Then, during a time interval \( \Delta t \), the external field is steadily reduced to half its original strength. Point P is located on the ground a distance \( x \) outside the loop, as shown.

Find the total magnetic field (magnitude and direction) that exists at point P during the time interval \( \Delta t \).

The list of known values: \( L, W, R, B_{\text{earth}}, B_{\text{ext}}, \theta, \Delta t, x \)

I. \( B_{\text{earth}} \) is directed vertically downward at Magnetic North.

II. Find the magnetic flux in the loop at the start of \( \Delta t \) (using a downward normal for the loop):

\[
\Phi_i = \left[ (B_{\text{earth}} L W) \cos(\phi_{\text{earth}}) + B_{\text{ext}} L W \cos(90 - \theta) \right]
\]

Solve for: \( \Phi_i \)

Where: \( B_{\text{earth}}, L, W, B_{\text{ext}}, \theta \) are all given

\( \phi_{\text{earth}} = 0^\circ \) (known)

III. Find the magnetic flux in the loop at the end of \( \Delta t \) (again, using a downward normal for the loop):

\[
\Phi_f = \left[ (B_{\text{earth}} L W) \cos(\phi_{\text{earth}}) + (B_{\text{ext}}/2) L W \cos(90 - \theta) \right]
\]

Solve for: \( \Phi_f \)

Where: \( B_{\text{earth}}, L, W, B_{\text{ext}}, \theta \) are all given

\( \phi_{\text{earth}} = 0^\circ \) (known)

IV. Find the magnitude of the voltage induced around the loop during the time interval \( \Delta t \):

\[
|\Delta V| = \left| \Delta \Phi/\Delta t \right| = \left| (\Phi_f - \Phi_i)/\Delta t \right|
\]

Solve for: \( |\Delta V| \)

Where: \( \Phi_f \) is from part III, and \( \Phi_i \) is from part II

\( \Delta t \) is given

V. Find the magnitude of the current induced around the loop during the time interval \( \Delta t \):

\[
I = (|\Delta V|)/R
\]

Solve for: \( I \)

Where: \( |\Delta V| \) is from part IV

\( R \) is given

VI. The induced current, \( I \), is directed clockwise, by Lenz’s Law.

VII. Find the field strength due to the current loop \( I \), as it appears at point P:

Only the near and far segments of the current loop \( I \) (i.e. the segments running along the width of the loop) will cause fields at point P; the field lines of the other two segments do not intersect point P. Therefore, using RHR #2 and defining upward (out of the earth) as the positive vertical direction…

\[
B_{P,\text{total}} = B_{P,\text{near}} - B_{P,\text{far}} = \mu_0 I/(2\pi x) - \mu_0 I/[2\pi(x+L)]
\]

Solve for: \( B_{P,\text{total}} \)

Where: \( \mu_0 \) is known

\( I \) is from part V

\( x \) and \( L \) are given

VIII. Find the total field at P due to both the current loop \( I \) and the earth (again, defining upward out of the earth as the positive vertical direction):

\[
B_{\text{total}} = B_{P,\text{total}} - B_{\text{earth}}
\]

Solve for: \( B_{\text{total}} \)

Where: \( B_{P,\text{total}} \) is from part VII

\( B_{\text{earth}} \) is given

(If the sign of \( B_{\text{total}} \) is positive, its direction is upward; if the sign of \( B_{\text{total}} \) is negative, its direction is downward.)