_prep_1

_Suggested finish date:_ Monday, April 8

The formats (type, length, scope) of these Prep problems have been purposely created to closely parallel those of a typical exam (indeed, these problems have been taken from past exams). _To get an idea of how best to approach various problem types (there are three basic types), refer to these sample problems._
1. a. Evaluate each statement (T/F/N). As always, you must fully explain your reasoning and answers.
   (i) An electrically charged object can exert a net electrical force on an electrically neutral object.
   True. The charged object can polarize the neutral object and thereby exert a net attraction force on the neutral
   object, because it will be more strongly attracted to its nearer side than it will be repelled by the farther side.
   (ii) An electrically neutral object can exert an electrical force on another electrically neutral object.
   True. If the first object is polarized, it will do likewise to the second object and the nearer ends will be more
   attracted than the farther ends will be repelled.
   (iii) An electrically neutral insulator can be polarized.
   True. If a charged object is brought near to a neutral insulator, the individual molecules of the insulator material
   will polarize “in place”—become slight dipoles, that then orient themselves with the opposite charge sign facing
   the nearby charged object. The macroscopic effect of all these molecular dipoles is to create a cumulative dipole
   over the entire object.
   (iv) If you place a charged object near to (but not touching) an unknown object, and there is a repelling
   force exerted by each on the other, the unknown object could be electrically neutral.
   Depends on your assumptions (so, N). If a charged object is held near a neutral object, the neutral object becomes
   polarized, so that the nearer end is of the opposite charge— which always attracts to the charged object (e.g.
   “static cling”). So, if you made the assumption that there are only two objects to consider, then repulsion could
   not occur. However, if there were a third object, also charged (same sign as the other) but more strongly, brought
   near the opposite end of the neutral object, that object’s greater charge would govern the polarization of the
   neutral object, so the neutral object would have to present its like-charge polarized end toward the weaker charged
   object, thus causing repulsion between those two.
   (v) A proton exerts a positively directed force on another proton.
   Not enough information. It all depends on the choice of coordinate axes. For example, if proton 1 is located at
   the origin and proton 2 is located along the positive x-, y- or z- axis, then \( F_{E,12} \) is indeed in a positive axial
   direction, and the statement is true. But if proton 2 is located along the negative x-, y- or z- axis (or at any point
   not on a coordinate axis), then \( F_{E,12} \) is not in a positive direction. The positive/negative designations of the axial
   directions have nothing to do with the positive/negative designations of charge.
   (vi) A proton exerts a positively directed force on an electron.
   Not enough information. The reasoning is similar to item (v), above.

b. X and Y are two uncharged metal spheres sitting at rest on adjacent insulating stands. The spheres are
   initially in contact with each other. A positively charged rod, R, is then brought close to X (without
   touching it). Sphere Y is then moved away from X. Then R is moved far away from both X and Y.
   What are the final net charges (positive, negative or neutral) of X and Y?
   As always, explain your answers—use drawings as needed to help you reason and explain.
   The two spheres initially in contact are, electrically speaking, one object, “XY.” When R is brought close to the
   X end, this polarizes XY, so that the X end is negatively charged, while the Y end is positively charged. Then
   separating the two spheres “strands” those net charges, respectively, so that X now carries a net negative charge; Y
   carries a net positive charge.
2. a. Explain briefly but fully: How does an object usually acquire a net positive charge?
   It loses electrons, which are either “scraped off” via friction (by a surface with a higher charge affinity) or “chased off” via polarization.

b. If a metal object becomes positively charged, does the mass of the object increase, decrease, or stay the same? Explain briefly but fully.
   Its mass decreases, because it has lost electrons which are loosely bound, especially in the outer valence orbitals. (The object has not gained protons, which are tightly bound in nuclei).

c. Explain briefly but fully: If a conductive object carries a net charge, how is the excess charge distributed on the object—and why is this so? Use one or more sketches if necessary.
   The excess charge resides on the outside of the object, because these charges are free to move anywhere (this is a conductor), and they repel one another. Thus, arrayed along the outside of the object is, by simple geometry, the farthest they can get from one another.

d. Explain briefly but fully: How can an electrically neutral object be attracted to a charged object? Use one or more sketches if necessary.
   The neutral object is polarized—see item 1a(i), above.

e. Explain briefly but fully: How can an initially neutral object acquire a positive charge without touching any charged object? Use one or more sketches if necessary.
   This would be the case of sphere Y in item 1b, above. (Y touched only X, which was neutral.)

f. How many electrons would you need in order to have 1.50 nC of negative charge? As always, show all work, reasoning, and calculations.
   \[
   \frac{-1.50 \times 10^{-9} \text{C}}{-1.60 \times 10^{-19} \text{C/electron}} = 9.38 \times 10^9 \text{ electrons}
   \]
3. a. Evaluate the following statement (T/F/N), and as always, fully explain your reasoning:
If you reduce by half the distance between two point charges, you double the magnitude of electrical force
that each exerts on the other.

b. Two point charges were previously 18.0 cm apart. Then they were moved so that the force on each has
become three times what it was before. How far apart are the charges now?

c. A point charge \( q_1 = +2e \) is located at (0, 0). Another point charge \( q_2 = -3e \) is located at (−2,0). A third
point charge \( q_3 = -6e \) is located at (3,0). Find the direction of the total electrical force exerted on \( q_1 \).

d. Let \( q > 0 \) and \( d > 0 \).
   A particle with charge \(+q\) is stationary and
   located at the point \( (0, d) \).
   A particle with charge \( -q \) is stationary and
   located at the point \( (0, -d) \).
   In which direction would an electron accelerate
   if it were located at \( (L, 0) \), where \( L > 0 \)?

e. Evaluate (T/F/N) the following statement. Justify your answer fully with any valid mix of words, drawings and
calculations. Three protons are located in the \( x\)-\( y \) plane: \( q_A \) is at \( (0, 0) \); \( q_B \) is at \( (1, 0) \); \( q_c \) is at \( (0, d) \). If the
net force on \( q_A \) is in the \( \angle 150^\circ \) direction (angle measured conventionally), then \( d \approx -1.32 \).

f. A point charge, \( q_1 \), is located at the origin, \( (0, 0) \). It exerts a force of known magnitude \( F_{12} \) on an electron,
\( q_2 \), located at the point \( (4,0) \). Write an expression for the magnitude \( F_{13} \) of the force that \( q_1 \) exerts on
another charge, \( q_3 = +2e \), located at \( (0, 2) \).

g. A point charge is located at the origin. It exerts a force of known magnitude \( F_{12} \) on an electron \( (q_2) \) located
at the point \( (4,0) \). Write an expression for the magnitude \( F_{13} \) of the force it exerts on a charge \( q_3 = -3e \),
located at \( (0,1) \).
4. a. Four identical particles of charge \( +Q \) are equally spaced along a horizontal line. The distance between adjacent charges is \( d \). What is the magnitude of the electric force on the charge on the far left end?

b. Three point charges, \( q_1, q_2 \) and \( q_3 \), are located along the \( x \)-axis, with the same distance \( d \) between \( q_1 \) and \( q_2 \) as between \( q_2 \) and \( q_3 \). \( q_2 = -3.40 \) nC. Find the value of \( q_1 \) that will put \( q_3 \) into static equilibrium.

c. Three charged particles are positioned along a line. Particle B is positioned a distance \( d \) to the right of particle A. Particle C is positioned an equal distance \( d \) to the right of particle B. Particles A and C are electrons. Particle B is a proton. If the magnitude of the net electrical force on particle C is \( 5.00 \times 10^{-25} \) N, calculate \( d \).

d. Point charges are fixed at the corners of an equilateral triangle, as shown. \( q_B \) is located at \( x = 2.50 \) cm. The value of \( q_C \) is known: \( q_C = +1.80 \) \( \mu \)C, and the net electrical force on \( q_C \) is in the negative \( y \)-direction and has a magnitude of 450 N. Find the magnitudes and signs of \( q_A \) and \( q_B \).
e. The current standard model of particle physics states that a proton is comprised of two “up” quarks, each of charge \((\frac{2}{3})e\), and one “down” quark, of charge \((-\frac{1}{3})e\). Assume that all three quarks are equidistant from each other at the distance of \(1.50 \times 10^{-15}\) m.

(i) What is the total force magnitude exerted on one of the “up” quarks by the other two quarks?

(ii) What is the total force magnitude exerted on the “down” quark by the other two quarks?

In the diagram, let \(q_c\) be the “down” quark; \(q_A\) and \(q_B\) are the “up” quarks.

(i) \(F_{E,\text{Total}} = F_{E,BA} + F_{E,CA}\)

\[
F_{E,\text{Total}} = k|q_B||q_A|/d^2 \angle 180^\circ + k|q_C||q_A|/d^2 \angle 60^\circ
\]

\[
= (4/9)k e^2/d^2 \cos(180^\circ, \sin(180^\circ)) + (2/9)k e^2/d^2 \cos(60^\circ, \sin(60^\circ))
\]

\[
= [(2/9)k e^2/(2d^2) - (8/9)k e^2/(2d^2), \sqrt{3}(2/9)k e^2/(2d^2)]
\]

\[
= [ke^2/(2d^2)][-2/3, (2\sqrt{3}/9)]
\]

Therefore:

\[
F_{E,\text{Total}} = [ke^2/(2d^2)][-2/3, (2\sqrt{3}/9)]^{1/2}
\]

\[
= [ke^2/(2d^2)][4/9 + 12/81]^{1/2}
\]

\[
= [ke^2/(2d^2)][36/81 + 12/81]^{1/2}
\]

\[
= [ke^2/(2d^2)][48/81]^{1/2}
\]

\[
= 2\sqrt{3}ke^2/(9d^2)
\]

\[
= 2\sqrt{3}(8.99 \times 10^{-9})(1.60 \times 10^{-19})^2/[9(1.50 \times 10^{-15})^2] = 39.4 \text{ N}
\]

(ii) \(F_{E,\text{Total}} = F_{E,AC} + F_{E,BC}\)

\[
= k|q_A||q_C|/d^2 \angle 240^\circ + k|q_B||q_C|/d^2 \angle 120^\circ
\]

\[
= (2/9)k e^2/d^2 \cos(240^\circ, \sin(240^\circ)) + (2/9)k e^2/d^2 \cos(300^\circ, \sin(300^\circ))
\]

\[
= (2/9)k e^2/d^2[-(1/2), -\sqrt{3}/2] + (2/9)k e^2/d^2[(1/2), -(\sqrt{3}/2)]
\]

\[
= [0, -(\sqrt{3}/2)ke^2/d^2]
\]

Therefore:

\[
F_{E,\text{Total}} = \sqrt{3}(2/9)ke^2/d^2
\]

That is:

\[
F_{E,\text{Total}} = F_{E,\text{Total}} = 39.4 \text{ N}
\]
5. a. Evaluate each statement (T/F/N). As always, you must fully explain your reasoning and answers.
   (i) Assuming no other charges (or fields) in the area, the electric field midway between a proton and electron is zero.
       False. See item 6b.

   (ii) The magnitude of the electric field caused by a point charge is the same at any point on a circle whose center is occupied by that charge.
       True. The E-field strength depends (spatially) only on the radial distance from the charge to the point in question: \( |E| = k|q|/r^2 \)

   (iii) The electric field is uniform in the region surrounding a single point charge.
       False. “Uniform” means “identical at every point.” The E-field surrounding a point charge varies in both magnitude (closer points vs. farther) and direction (radially outward can mean any \( \theta \) and \( \phi \) angles of spherical coordinates). Don’t confuse “uniform” with “symmetric.” The E-field around a point charge is indeed spherically symmetric (looks similar even after rotating), but the vector values at many points are definitely different.

b. An electron is located at the origin. Another electron is located at (3,0). Find the E-field direction at (1,0), using the positive x-axis as 0°.

c. An electron is located at the origin. A proton is located at (2,0). Find the E-field direction at (–1,0).

d. An electron is located at the origin. A proton is located at (2,0). Find the electric field direction at (1,1).

e. What’s the direction of the electric field caused at the point (–2,3) by an electron located at the origin?
6. a. The net electric field produced by two charges is shown to the right. Fully explain your reasoning and answer for each of the following:

(i) What can you conclude about each of the two charges, \( P \) and \( Q \)?

\( P \) is the charge on the left; \( Q \) is on the right, and each charge’s field lines emerge from it. Sorry for the poor resolution here.)

Both \( P \) and \( Q \) are positive charges; their field lines radiate outward. And \( Q > P \); \( Q \) creates more field lines at any given radial distance from it (this is used to represent a stronger field).

(ii) Where in the diagram is the electric field magnitude strongest — and where it is weakest?

Again, a greater number of field lines shows a greater total field strength, so very near \( Q \) (on sides not next to \( P \)) would be areas of strongest fields; the region midway between \( P \) and \( Q \) would be areas of weakest fields.

(iii) How does the force of \( P \) on \( Q \) compare to the force of \( Q \) on \( P \)?

Newton’s third law applies to any kind of force. Thus:

\[
F_{E.PQ} = \mathbf{–F_{E.QP}}
\]

The force magnitudes are equal; the force directions are opposite.

b. In the drawing, \( q_1 \) is located at point A, and \( q_2 \) is located at point B.

Let \( E_{1B} \) mean “the electric field caused at point B by charge 1;” and let \( E_{2A} \) means “the electric field caused at point A by charge 2.”

Evaluate each statement (T/F/N). As always, you must fully explain your reasoning and answers.

(i) If both \( q_1 \) and \( q_2 \) are negative, the direction of \( E_{2A} \) is the same as the direction of \( F_{12} \).

True. If both charges are negative, then:

\[
E_{2A} = (k|q_2|/d_{AB}^2) \angle 0^\circ \quad \text{(The field vector at A points toward } q_2 \text{ — that’s to the right.)}
\]

\[
F_{12} = (k|q_1|/d_{AB}^2) \angle 0^\circ \quad \text{(} q_1 \text{ repels } q_2, \text{ pushing it to the right)}
\]

(ii) Regardless of their signs (±), if \( q_1 \) and \( q_2 \) are of equal magnitude, then \( E_{2A} = \mathbf{–E_{1B}} \), always.

False. Suppose that \( q_1 \) is positive and \( q_2 \) is negative. Then the two vectors are identical (not opposites):

\[
E_{2A} = (k|q_2|/d_{AB}^2) \angle 0^\circ \quad \text{(The field vector at A points toward } q_2 \text{ — that’s to the right.)}
\]

\[
E_{2A} = (k|q_1|/d_{AB}^2) \angle 0^\circ \quad \text{(The field vector at B points away from } q_1 \text{ — that’s also to the right.)}
\]

c. Write an expression for the E-field magnitude midway between an electron and proton that are a distance \( d \) apart.
6. d. Two electrons are located along the y-axis at points (0, s) and (0, –s).
You may consider the following as known values (so they may be included in your answers): \(k, \varepsilon_0, e, s\)

(i) Write an expression for the electric force magnitude exerted on one electron by the other.

The basic Coulomb force equation: \(|F_{E,ij}| = k|q_i||q_j|/r_{ij}^2\)
In this situation: \(|q_i| = |q_j| = e\)
And: \(r_{ij} = 2s\)
Thus: \(|F_{E,ij}| = ke^2/(2s)^2 = ke^2/(4s^2)\)

(ii) Write an expression for the net electric field (magnitude and direction) at the point \((r, 0)\) on the positive x-axis. (You may consider \(r\) as a known value, too, so your answer may include \(r, k, \varepsilon_0, e, s\).) Be sure to use diagrams if they help!
First, note from the diagram that only the x-components of each field will be significant; due to the symmetry of the situation, their y-components will cancel each other. We need only \(E_x\). So we need only \(E_{1x}\) and \(E_{2x}\).

The basic field equation: \(|E_i| = k|q_i|/r_i^2\)
Take the x-component only: \(E_{1x} = -(k|q_i|/r_i^2)(\cos \theta) = -(k|q_i|/r_i^2)(\cos \theta)\)
In this situation: \(r_i = r_2 = \sqrt{r^2 + s^2}\)
And: \(\cos \theta = r/r_i\)
And, again: \(|q_i| = |q_j| = e\)
Therefore: \(E_{1x} = -(ker/r_i^3) = -(ker)/(r^2 + s^2)^{3/2}\)
Double this to add \(E_{2x}\): \(E_x = -(2ker)/(r^2 + s^2)^{3/2}\)

(iii) Referring to the field expression from part b, how would you calculate the value of \(r\) where (along the x-axis) that field strength is a maximum? *You do not have to actually do the calculation; just fully explain your reasoning and the procedure.*

The expression from part b is a function, \(E_x(r)\). To find a local maximum, we would therefore take the derivative of that function and set it equal to zero:
\[
dE_x/dr = 0 \quad \text{In other words, we’d solve} \quad d(-(2ker)/(r^2 + s^2)^{3/2})/dr = 0 \quad \text{for} \ r.
\]