Midterm Exam 1

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Prob. 1: _______
Prob. 2: _______
Prob. 3: _______
Prob. 4: _______
Adjustment: _______
Appeal: _______
TOTAL: ________/ 250
1. (60 points total) A line segment of uniform positive charge density \( \lambda \) is placed at rest on the \( x \)-axis. Its end points are located at \((0, 0)\) and \((L, 0)\), where \( L \) is a positive value.

Using only known values, set up an integral that would correctly evaluate the total electric field, both magnitude and direction, at the point \((-L, 0)\). Do not evaluate the integral but prepare it fully for evaluation, showing the correct direction signs and the correct limits of integration.

**Here are known values:** \( \lambda, L, k, \varepsilon_0 \)

Let the point \((-L, 0)\) be called point \( P \). Then note first that there is no \( E_y \) at \( P \), because all field acts along the line between each bit of charge and the point \( P \) — and that line in this case is always the \( x \)-axis.

In other words: \( E_{x,P} = 0 \)

Therefore: \( E_{\text{total},P} = E_{\text{total},Px} \)

So we need to set up the integral to sum up \( E_{x,P} \) (and a diagram such as the one above is a very good idea).

For any arbitrarily chosen bit of charge, \( dq \), along the line segment, its \( x \)-component contribution, \( dE_x \), at point \( P \) is given by the Coulomb equation...

\[
\text{...like this: } \quad dE_{x,P} = -k(dq)/(r)^2 \\
\text{where: } \quad r = x + L \quad \text{(note the geometry from the diagram)}
\]

Substituting:
\[
dE_{x,P} = -k(dq)/(x + L)^2 \quad \text{4 pts.}
\]

But \( dq = \lambda dx \):
\[
dE_{x,P} = -k(\lambda dx)/(x + L)^2 \quad \text{10 pts.}
\]
\[
= -k\lambda [1/(x + L)^2]dx \quad \text{4 pts.}
\]

And to get \( E_{\text{total},Px} \), we need to sum this contribution for every \( dq \) located throughout the \( x \)-range of \( 0 \leq x \leq L \):

Like this:
\[
E_{\text{total},Px} = \int dE_x = -k\lambda \int_0^L [1/(x + L)^2]dx
\]

Therefore:
\[
E_{\text{total},P} = -k\lambda \int_0^L [1/(x + L)^2]dx \quad 6 \text{ points total: expression = 3; limits = 2; sign = 1}
\]
Use this page as additional space, if needed, for problem 1
2. **(60 points)** Three point charges are fixed in place in a space marked with a set of x-y axes (all coordinates given here in m).

Let $\angle 0^\circ$ indicate the $+x$-direction and $\angle 90^\circ$ the $+y$-direction.

Charges $q_A$ and $q_B$ are protons.
- $q_A$ is located at the origin.
- $q_B$ is located at point $(3, 0)$.

Charge $q_C$, located at point $(3, 3)$, is unknown.

The total electric field at the empty point $(10^{-5}, 0)$ is approximately $(ke \times 10^{-10})$ N/C $\angle 0.00^\circ$, where $k$ is the Coulomb constant, and $e$ is the charge on a proton.

Calculate the total force (magnitude and direction) exerted on $q_C$ by the other two charges.

You may choose to express the values of $e$, $k$ and/or $\varepsilon_0$ either as numbers or as symbols.

First, to the final clue...

Any collection of charge will produce an electric field “far away” that mathematically approximates the field of a point charge—the net charge of the collection.

The electric field at that large distance (100 km) resembles that of a single proton; its E-field magnitude is correct for that, AND its direction is away from the origin.

**Conclusion:** The net charge of the three charge collection must be that of a single proton ($+e$). So the charge on $q_C$ must be $-e$.

(It’s probably an electron, but we don’t know, nor care about, its mass.)

Now it’s just a summing of forces on $q_C$:

It’s a vector sum: 

\[
F_{E,\text{total},C} = F_{E,AC} + F_{E,BC}
\]

\[
= F_{E,AC} \angle 225^\circ + F_{E,BC} \angle 270^\circ \quad \text{(both are attraction forces on } q_C)\]

\[
= (ke^2/18)\angle 225^\circ + (ke^2/9)\angle 270^\circ \quad \text{(Coulomb force magnitudes)}\]

In rectangular form:

\[
F_{E,\text{total},C} = \begin{bmatrix}
(ke^2/18)\cos225^\circ, (ke^2/18)\sin225^\circ \\
(ke^2/9)\cos270^\circ, (ke^2/9)\sin270^\circ
\end{bmatrix}
\]

\[
= \begin{bmatrix}
(ke^2/18)\cos225^\circ, (ke^2/18)\sin225^\circ - 2 \\
(ke^2/18)\sin225^\circ - 2
\end{bmatrix}
\]

\[
= (ke^2/18)[-0.70711, -2.70711]
\]

Total magnitude:

\[
F_{E,\text{total},C} = (ke^2/18)\sqrt{(-0.70711)^2 + (-2.70711)^2} = (0.155441)ke^2 \quad 2 \text{ pts.}
\]

Direction:

\[
\theta_{FE,\text{total},C} = \tan^{-1}(-2.70711/(-0.70711)) + 180^\circ \quad (\theta_{FE,\text{total},C} \text{ is in Quadrant III!}) \quad 8 \text{ pts.}
\]

\[
= 255.4^\circ \quad 2 \text{ pts.}
\]

Therefore:

\[
F_{E,\text{total},C} = (0.155)ke^2 \text{ N } \angle 255^\circ \quad (\text{or } \angle -105^\circ) \quad 6 \text{ points total: value = 2; angle = 2; units =1; sig. figs. = 1}
\]

or:

\[
F_{E,\text{total},C} = 3.58 \times 10^{-29} \text{ N } \angle 255^\circ \quad (\text{or } \angle -105^\circ)
\]
3. (60 points total) Two thin rings of uniformly distributed net charge are placed at rest concentrically—so that they share a common center (at the origin) and a common central axis (the z-axis), as shown in this “edge-on” view.

Point P is located along the z-axis a distance R from the origin, and the total electric field at point P is zero.

The larger ring (A) has radius R and linear charge density \( \lambda_A = +1.20 \text{ nC/m} \).

The smaller ring (B) has radius \( R/2 \).

Calculate \( \lambda_B \) (its numeric value and units).

Clearly, the smaller ring must have negative charge; otherwise its field at P could never sum with the larger ring’s field to make zero. So their fields at point P are opposite in sign, but equal in magnitude.

That is: \( E_{AP_z} = -E_{BP_z} \)

In more detail: \( k|Q_A|/[z^2 + R^2]^{3/2} = k|Q_B|/[z^2 + (R/2)^2]^{3/2} \)

But in this case, \( z = R \):

\[ kR|Q_A|/[2R^3]^{3/2} = kR|Q_B|/[(5/4)R^3]^{3/2} \]

Simplify:

\[ |Q_A|/2^{3/2} = |Q_B|/(5/4)^{3/2} \]

For any ring of uniformly distributed charge...

We know: \( |Q| = (2\pi R)|\lambda| \)

So in this case: \( (2\pi R)|\lambda_A|/2^{3/2} = (2\pi R/2)|\lambda_B|/(5/4)^{3/2} \)

More simplifying: \( |\lambda_A|/2^{3/2} = (1/2)|\lambda_B|/(5/4)^{3/2} \)

Solve for \( |\lambda_B| \):

\[ |\lambda_B| = 2|\lambda_A|(5/8)^{3/2} \]

\[ = 2(1.20 \times 10^{-9})(5/8)^{3/2} = 1.186 \times 10^{-9} \]

Therefore: \( \lambda_B = -1.19 \times 10^{-9} \text{ C/m} \) or \( -1.19 \text{ nC/m} \) (either units OK)
Use this page as additional space, if needed, for problem 3
4. **(70 points)** Suppose you have an infinitely wide slab of non-conducting material that is of known thickness \(D\). This slab has a net charge that is distributed uniformly throughout it.

You perform the following experiments:

**Experiment A:** You temporarily remove all the slab except for a cube whose sides are all of known length \(s\) \((s \leq D)\). You place this cube with its center at the origin and you define a spherical Gaussian surface around it. That spherical Gaussian surface is centered at the origin and has a radius of \(2s\).

**Result:** The net flux through this surface is a known positive value, \(\Phi_{\text{cube}}\).

**Experiment B:** You restore the slab completely—including the cube of charge that you had extracted. You release an electron (from rest) at a distance \(H\) above the slab’s surface.

**Result:** The electron (mass \(m_e\), charge \(e\)) reaches the slab surface in a time \(\Delta t\).

**Write an expression for the time \(\Delta t\).** Your expression may contain only known values.

These are known values: \(D\), \(s\), \(\Phi_{\text{cube}}\), \(H\), \(m_e\), \(e\), \(k\), \(e_0\)

**Analysis of Experiment A:**

Gauss’s Law applies to any closed surface, so it applies here.

The basic equation: \[ \Phi_{\text{E, closed}} = \frac{Q_{\text{enclosed}}}{\varepsilon_0} \]

In this case: \[ \Phi_{\text{cube}} = \frac{Q_{\text{cube}}}{\varepsilon_0} \] \(\text{(Defined sphere encloses the entire cube.)}\)

Solving for \(Q_{\text{cube}}\).

\[ Q_{\text{cube}} = \Phi_{\text{cube}} \varepsilon_0 \]

Cube’s charge density is uniform:

\[ \rho_{\text{cube}} = \frac{Q_{\text{cube}}}{V_{\text{cube}}} \]

Substitute:

\[ \rho_{\text{cube}} = \frac{\Phi_{\text{cube}} \varepsilon_0}{s^3} \]

Cube was taken from slab:

\[ \rho_{\text{slab}} = \frac{\Phi_{\text{cube}} \varepsilon_0}{s^3} \]

**Analysis of Experiment B:**

To determine how the electron moves, we need the electric field strength in the space between the slab and the electron. Start by calculating the field at the electron’s release point.

Define a Gaussian surface as a box (or cylinder—either works) whose sides are perpendicular to the slab and whose ends are parallel to the slab. The ends have identical areas \(A\), and these ends are located on opposite sides of the slab. Here, for example, they are the same distance \(H\) from one side of the slab (“edge-on” view).

- The slab is infinitely wide, so we know that all electric field is perpendicular to it, so the sides of the box will have no flux through them.
- The slab’s charge distribution is symmetric—*it looks the same if viewed from either side*—so we know that the field through each of the ends will be the same value \(E\) (directed away from the slab’s positive charge).

So Gauss’s Law is very handy here....
Gauss’s Law: \[ \Phi_{\text{box}} = \frac{Q_{\text{enclosed}}}{\varepsilon_0} \] 
Only the ends have flux: \[ 2EA = \frac{Q_{\text{enclosed}}}{\varepsilon_0} \] 
Slab has uniform charge density: \[ Q_{\text{enclosed}} = \rho_{\text{slab}} V_{\text{enclosed}} \] 
where... \[ V_{\text{enclosed}} = AD \] 
Substitute: \[ 2EA = \rho_{\text{slab}} AD/\varepsilon_0 = (\Phi_{\text{cube}} \varepsilon_0/s^3)AD/\varepsilon_0 = \Phi_{\text{cube}} AD/s^3 \] 
Solve for \( E \): \[ E = \Phi_{\text{cube}} D/(2s^3) \] 

Notice that the electron will experience the same force throughout its “fall”—so we can use constant-acceleration kinematics.

The force: \[ F = eE = e\Phi_{\text{cube}} D/(2s^3) \] 
The acceleration: \[ a = F/m_e = e\Phi_{\text{cube}} D/(2m_e s^3) \] 
The kinematics: \[ H = (1/2)a(\Delta t)^2 \] (electron released from rest; falls a distance \( H \)) 
Solve for \( \Delta t \): \[ \Delta t = \sqrt{2H/a} \] 
Substitute for \( a \): \[ \Delta t = \sqrt{[4Hm_e s^3/(e\Phi_{\text{cube}} D)]} \]
Physical constants and other possibly useful information:

\[ k = 8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 \]
\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2) \]
\[ e = 1.60 \times 10^{-19} \text{ C} \]
\[ m_e = 9.11 \times 10^{-31} \text{ kg} \]
\[ \Delta x = v_{ix} \Delta t + (1/2)a_x \Delta t^2 \]
\[ v_{fx} = v_{ix} + a_x \Delta t \]
\[ v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \]
\[ \sqrt{E_{z,\text{axis}}} = \frac{kz|Q|}{(z^2 + R^2)^{3/2}} \]