Prep 9

*Suggested finish date:* Monday, March 11

The formats (type, length, scope) of these Prep problems have been purposely created to closely parallel those of a typical exam (indeed, these problems have been taken from past exams). *To get an idea of how best to approach various problem types (there are three basic types), refer to these sample problems.*
1. a. Evaluate the following statements (T/F/N). As always, explain your reasoning.

(i) If a standing sound wave in a tube open at both ends is vibrating at its 3rd harmonic frequency, there is a displacement node at its midpoint.
   True. At either open end is a displacement antinode, and the tube has a length of \(3\lambda/2\) (or \(6\lambda/4\)), so the displacement graph would have the following features, spaced in intervals of \(\lambda/4\):
   \[\text{AN} – N – \text{AN} – N – \text{AN} – N – \text{AN}\] (The midpoint is an N.)

(ii) The fundamental frequency of a standing sound wave in a tube open at both ends is higher when the air is warmer.
   True. \(f_1 = (1)v/(2L)\), and the speed, \(v\), of the sound wave increases with temperature.

(iii) If two guitar strings of equal length vibrate at fundamental frequencies that are one octave apart, they cannot have the same tension.
   False. \(f_1 = (1)v/(2L)\), where \(v = \sqrt{(F/T)/\mu}\). If \(f_{1B} = 2f_{1A}\) (that’s an octave apart), this means that \(v_B = 2v_A\). This is possible even if \(F_{TA} = F_{TB}\). It would mean that \(\mu_A = 4\mu_B\).

(iv) If there is a standing sound wave in a tube open at both ends, and it is vibrating at its fundamental frequency, there is a pressure antinode at the midpoint (halfway between the ends).
   True. At either open end is a pressure node, and the tube has a length of \(\lambda/2\) (or \(2\lambda/4\)), so the pressure graph would have the following features, spaced in intervals of \(\lambda/4\):
   \[N – \text{AN} – N\] (The midpoint is an AN.)

(v) A string of length \(L\), fixed at both ends, and a tube of the same length \(L\), open at both ends, must have the same harmonic frequencies.
   False. \(f_1 = (1)v/(2L)\), where \(v\) is the speed of the wave. The wave speed on the string is not necessarily (indeed, rarely) the same as the speed of sound in air.

(vi) A string of length \(L\), fixed at both ends, and a tube of the same length \(L\), open at one end, both resonating at their 3rd harmonic frequency, must have the same number of displacement antinodes.
   False. The string has a length of \(3\lambda_S/2\) (or \(6\lambda_S/4\)), so its displacement graph would have the following features, spaced in intervals of \(\lambda_S/4\):
   \[N – \text{AN} – N – \text{AN} – N – \text{AN} – N\] There are 3 anti-nodes here.
   The tube has a length of \(3\lambda_T/4\), so its displacement graph would have the following features, spaced in intervals of \(\lambda_T/4\):
   \[\text{AN} – N – \text{AN} – N\] There are 2 anti-nodes here.

(vii) Not counting the two fixed ends, the 2nd resonant frequency of a violin string has two displacement nodes.
   False. The string has a length of \(2\lambda/2\) (or \(4\lambda/4\)), so its displacement graph would have the following features, spaced in intervals of \(\lambda/4\):
   \[N – \text{AN} – N – \text{AN} – N\] Not counting the ends, there is just 1 node here.

(viii) If a tube of air is open at one end, closed at the other, and is vibrating at its 3rd harmonic frequency, there is a pressure node at the tube’s midpoint.
   False. The tube has a length of \(3\lambda/4\), so its pressure graph would have the following features, spaced in intervals of \(\lambda/4\):
   \[N – \text{AN} – N – \text{AN}\] The midpoint is neither a node nor an antinode.

(ix) If there is a standing wave on a string that is fixed at both ends, the length of the string is always a multiple of half-wavelengths.
   True. This is the basic constraint that leads to resonance: \(L = m(\lambda/2)\), where \(m = 1, 2, 3, \ldots\)

(x) In a standing wave, the distance between a node and an antinode is always a multiple of half-wavelengths.
   False. The distance between a node and its adjacent antinode is just one quarter-wavelength.

(xi) For a guitar string (fixed at both ends, with constant length, tension and linear mass density), its 3rd harmonic frequency is 2 octaves higher in pitch than its fundamental frequency.
   False. \(f_1 = (1)v/(2L)\), and \(f_3 = (3)v/(2L)\), so \(f_3/f_1 = 3\). A two-octave interval would be a ratio of 4:1, since each octave represents a doubling of pitch (thus \(f_4\) would be two octaves above the fundamental; \(f_8\) would be three octaves above; etc.).
1. b. A certain string (fixed at both ends) will vibrate in a standing wave at a frequency of 113 Hz. The speed of the waves along the string is 270 m/s. How far apart are the nodes?
   Nodes are always λ/2 apart, and λ = v/f, so the node-node distance is λ/2 = v/(2f) = 270/(2·113) = 1.19 m

c. The 4th harmonic standing wave in an organ pipe—open at both ends—has a frequency of 1400 Hz and a speed of 350 m/s. What is the length of the organ pipe?
   \[ f_4 = (4)v/(2L), \text{ so } L = 4v/(2f_4) = 4(350)/(2·1400) = 0.500 \text{ m} \]

d. A musical wind instrument produces the following frequencies and no other frequencies in between: 125 Hz, 375 Hz, 625 Hz and 875 Hz. The speed of sound is 375 m/sec. How long is the instrument?
   The frequencies given here have ratios of 875:625:375:125 = 7:5:3:1. So they must be harmonics, and since there are no other frequencies between (only these odd harmonics), this wind instrument must be a tube open at one end but closed at the other. So, for any harmonic frequency:
   \[ f_m = mv/(4L) \]
   Thus we know (for example): \( f_7 = (7)v/(4L) \)
   Thus: \( L = (7)v/(4f_7) = 7(375)/(4·875) = 0.750 \text{ m} \)

e. Two strings, A and B, have equal lengths, but unequal linear densities. They are tied end-to-end with a knot and stretched between two supports. A particular frequency happens to produce a standing wave on each length, with a node at the knot. If B is resonating at its fundamental frequency, but A is at its 2nd harmonic frequency, which string has the greater linear density? As always, explain/justify this fully.
   \[ f_{2A} = (2)v_A/(2L_A) \]
   And \( f_{1B} = (1)v_B/(2L_B) \)
   But \( f_{2A} = f_{1B} \) (and \( L_A = L_B = L \)).
   So: \( (2)v_A/(2L) = (1)v_B/(2L) \)
   Thus: \( 2v_A = v_B \)
   Or: \( 2\sqrt{F_T/\mu_A} = \sqrt{F_T/\mu_B} \)
   Or: \( 4F_T/\mu_A = F_T/\mu_B \)
   Thus: \( \mu_A = 4\mu_B \)
   String A has the greater linear mass density.

f. If the speed of sound in air is 343 m/s, what is the third-lowest frequency at which an empty bottle of length 44.0 cm will resonate when you blow across its open end?
   For this bottle (a tube open at one end but closed at the other), only the odd harmonics are possible, so the third lowest resonant (harmonic) frequency is \( f_3 \):
   \[ f_3 = (5)v/(4L) = 5(343)/(4·0.44) = 974 \text{ Hz} \]

g. A standing wave on a string (fixed at both ends) was initially a traveling wave that could be described by \( y = 3.5\cos[3.14(x) – 2,765(t)] \), where the position along the string is measured in m, time in s, and the amplitude in mm. The string has a mass of 617 mg. Assuming that this string is now vibrating at its fundamental frequency:
   (i) How long is the string? For \( f_1, L = (1)\lambda/2 \), where 3.14 = 2π/\( \lambda \).
   Thus: \( L = (1/2)(2\pi/3.14) = 1.00 \text{ m} \)
   (ii) What is the tension in the string?
   \[ \omega/k = v = \sqrt{F_T/(m/L)}, \text{ so } F_T = (\omega/k)^2(m/L) = (2765/3.14)^2(0.000617/1.00) = 478 \text{ N} \]

h. A string fixed at both ends is vibrated, resulting first in a traveling wave that could be described by \( y = A\cos(15x – 2764t) \). Then, as this wave resounds back and forth along the string, it builds into a standing wave vibrating at the string’s 2nd harmonic frequency, with an amplitude at the antinodes of 2.0 mm.
   (i) How long does the wave take to travel the string’s length?
   For \( f_2 \), \( L = (2)\lambda/2 = \lambda \). A wave travels 1 wavelength in 1 period, \( T = 2\pi/2764 = 2.27 \times 10^{-3} \text{ s} \)
   (ii) What is the average speed of one particle on that string, as calculated over one complete cycle—assuming that the particle is located at an antinode?
   A particle oscillating travels 4 amplitudes in each period, so: \( v_{avg\ particle} = 4(.002)/(2.273 \times 10^{-5}) = 3.52 \text{ m/s} \)
2.  a. You have three identical, empty beverage bottles, A, B and C (each open at the top and closed at the bottom), each with an inside length of 25 cm. Assume the speed of sound in air is 344 m/s. Using the least water possible, add whatever water you judge necessary to any bottle (A gets the least water; C gets the most), so that when air is blown across all the bottles, their fundamental frequencies produce a major chord (i.e. frequency ratios of 4:5:6). For each bottle, what is the required water level (as measured in cm from the bottom of the bottle)?

b. Astronauts visiting Planet X have a 2.5 m long string whose mass is 5 g. They tie the string to a support, stretch it horizontally over a pulley 2 m away, and hang a 1 kg mass on the free end. Then they attempt to produce standing waves on the string. They are successful for vibration frequencies of 64 Hz and 80 Hz but at no frequencies in between. What is “g” on planet X?
2. c. It is observed that a standing sound wave will vibrate at a frequency of 440 Hz in a certain tube that is open at both ends. When one end of the tube is closed, a standing sound wave will vibrate at a frequency of 330 Hz. The speed of sound is 343 m/s. Find the minimum possible tube length.

In a tube open at both ends, there will be a standing sound wave only if the length of the tube, \( L \), can exactly accommodate a whole number of half-wavelengths:

\[ L = n(\lambda/2), \text{ where } n = 1, 2, 3,... \]

Rearranging this (and using \( \lambda = v/f \)), we get:

\[ f_n = nv/2L, \text{ with } n = 1, 2, 3, ... \]

where \( v \) is the speed of sound in that air.

In a tube open at just one end, there will be a standing sound wave only if the length of the tube, \( L \), can exactly accommodate an odd number of quarter-wavelengths:

\[ L = n(\lambda/4), \text{ where } n = 1, 3, 5,... \]

Rearranging this (and using \( \lambda = v/f \)), we get:

\[ f_n = nv/4L, \text{ with } n = 1, 3, 5, ... \]

again, where \( v \) is the speed of sound in that air.

In this problem, the length of the tube is the same in each case, so we can equate the expression for \( L \) from each equation above (denoting the first case as A and the second case as B):

\[ L = n_Av/2f_A = n_Bv/4f_B. \]

Rearranging to express \( n_B/n_A \), we get this:

\[ n_B/n_A = 2f_B/f_A = 2(330)/440 = 1.5 \]

So, “What integers \( n_A \) and \( n_B \) would have such a ratio?” (And \( n_B \) must be an odd integer, remember.)

There are an infinite number of possibilities, such as:

\[ n_B = 3, \quad n_A = 2 \]
\[ n_B = 9, \quad n_A = 6 \]
\[ n_B = 27, \quad n_A = 18 \]

... etc.

But we’re looking for the \( n \)-values that would imply the shortest tube length, \( L \), and since \( L \) is proportional to the \( n \)-value (in either case), we choose the minimum \( n \)-values: \( n_B = 3, \quad n_A = 2 \)

Using either of these values in its respective formula above will give \( L = 0.780 \text{ m} \).

d. A physics student is measuring the depth of a water well (which is basically an air-filled cylinder, closed at the bottom by the water surface. A sound tone generator, whose frequencies can be varied from 75.0 Hz to 150.0 Hz, produces resonances at 91.0 Hz, 117 Hz, and 143 Hz, but no other frequency in the 75-150 Hz range. Note: \( v_{\text{air}} = 331.3 + T_{\text{c}}/[0.610 \text{ m/s}^{\circ}\text{C}] \).

(i) What are the harmonic numbers of the three resonances?

(ii) If the air temperature in the well is 11.7 \(^{\circ}\text{C}\), what is the depth of the well?

(i) All harmonic frequencies are whole-number ratios of one another: \( f_m = mv/(4L) \), where \( m \) is an odd integer.

The frequencies given here have ratios of 143:117:91 = 11:9:7. So they must be \( f_1 \), \( f_2 \), and \( f_3 \), respectively.

(ii) Then we know:

\[ f_1 = (11)v/(4L) \]

That is:

\[ L = (11)v/(4f_1), \text{ where } v = 331.3 + (11.7)0.61 = 338.4 \text{ m/s} \]

Thus:

\[ L = (11)(338.4)/(4\cdot143) = 6.51 \text{ m} \]

e. Pipe A is 35 cm long, open at one end and closed at the other. Pipe B is four times as long as pipe A and open at both ends. The speed of sound in the air in both pipes is 350 m/s.

(i) What are the two longest standing wavelengths possible for each pipe?

Pipe A: \( L = m\lambda/4 \), where \( m \) is an odd integer. Thus: \( \lambda_1 = 4L/1 = 1.40 \text{ m} \); and \( \lambda_3 = 4L/3 = 0.467 \text{ m} \)

Pipe B: \( L = m\lambda/2 \), where \( m \) is an integer. Thus: \( \lambda_1 = 2L/1 = 2.80 \text{ m} \); and \( \lambda_2 = 2L/2 = 1.40 \text{ m} \)

(ii) What is the smallest frequency of a standing wave in pipe B which has the same frequency as one of the standing waves in pipe A?

The smallest common frequency is the longest common wavelength. Judging from the results of (i), this would have to be \( \lambda = 1.40 \text{ m} \). Thus, \( f = v/\lambda = 350/1.40 = 250 \text{ Hz} \)

(iii) What would happen to the lowest possible standing wave in the open-closed pipe if it were filled with a gas for which the speed of sound is 700 m/s? (Give a numeric comparison to the old condition.)

Pipe A: \( L = m\lambda/4 \), where \( m \) is an odd integer. Again, \( \lambda_1 = 4L/1 = 1.40 \text{ m} \). This is the same wavelength, but since \( v \) is twice as fast (\( f = v/\lambda \)), the frequency of the sound produced will be doubled (i.e. an octave higher): \( f_1 = 500 \text{ Hz} \)
2.  
You have a clarinet (a tube open at one end, closed at the other), a flute (open at both ends) and a piano string (fixed at both ends). The clarinet and flute are each resonating at their 3rd harmonic frequencies. The piano string is resonating at its fundamental frequency. The clarinet’s pitch is one octave below the flute’s. The piano string’s pitch is 2 octaves above the flute’s, but the piano string is only 10% as long as the flute. If the clarinet is 0.840 m long, and mass of the stretched piano string is just 1.00 g, what is the tension in that string? Assume standard atmospheric conditions here ($v_{\text{sound}} = 343$ m/s).

The clarinet:  
$f_{3,\text{clarinet}} = \frac{3v_{\text{sound}}}{4L_{\text{clarinet}}}  
= \frac{3(343)}{4(0.840)}  = 306.25 \text{ Hz}$

The flute:  
$f_{3,\text{flute}} = 2f_{3,\text{clarinet}} = 612.5 \text{ Hz}$  
So:  
$612.5 = \frac{3v_{\text{sound}}}{2L_{\text{flute}}}$  
Thus:  
$L_{\text{flute}} = \frac{3v_{\text{sound}}}{2(612.5)} = 0.840 \text{ m}$

The string:  
$f_{1,\text{string}} = 4f_{3,\text{flute}} = 2450 \text{ Hz}$  
So:  
$2450 = \frac{v_{\text{string}}}{2L_{\text{string}}}$  
Thus:  
$v_{\text{string}} = 2450(2L_{\text{string}})  
= 2450(2)(0.10)(0.840)  = 411.6$

So:  
$411.6 = \sqrt{\frac{F_{T,\text{string}}}{m_{\text{string}}/L_{\text{string}}}}$  
Thus:  
$F_{T,\text{string}} = (411.6)^2\left(\frac{m_{\text{string}}}{L_{\text{string}}}\right)  
= (411.6)^2(0.001)/(0.10)(0.840) = 2.02 \times 10^3 \text{ N}$

g.  
You have a piccolo (a tube open at both ends), a soprano saxophone (open at one end, closed at the other), and a cello string (fixed at both ends). The cello string and saxophone have the same length, which is 2.5 times the length of the piccolo. The piccolo is resonating at its fundamental frequency. The saxophone is sounding the same pitch as the piccolo. The saxophone’s harmonic mode ($m$ value) is the same as the cello string’s. The string’s linear mass density is 0.005 kg/m, and its tension is 2339.28 N. The string’s pitch, 2280 Hz, is the same as the piccolo’s 4th harmonic frequency. What is the speed of sound in this situation?
3. a. Light from a laser forms a 1.31 mm diameter spot on a wall. If the light intensity in the spot is 3.03 x 10^4 W/m^2, how much energy will the laser output in 10 s?

\[ P = I A \quad \text{And} \quad E = Pt \quad \therefore E = IAt = (3.03 \times 10^4)(\pi/4)(1.31 \times 10^{-3})^2(10) = 0.408 J \]

b. Express sound intensity in fundamental (base) SI units. Intensity (I) has units of W/m^2 = J/(s·m^2)

\[ = (N·m)/(s·m^2) = [(kg·m/s^2)·m]/(s·m^2) = (kg·m^2)/(s^3·m^2) = kg/s^3 \]

c. Hearing damage may occur when a person is exposed to 96.0-dB sound for a period of 8.50 hours. If an eardrum has an area of 2.00 x 10^-4 m^2, how much energy is incident on the eardrum over that time period?

\[ I = P/A, \quad \text{or} \quad P = IA \quad \text{And} \quad E = Pt \quad \therefore E = IAt \quad \text{We must calculate } I: \]

If \( \beta = 96 \) dB:

\[ 10·\log(I/I_o) = 96 \quad \text{Or:} \quad I/I_o = 10^{9.6} \quad \text{Or:} \quad I = 10^{9.6}(10^{-12}) = 10^{-2.4} \]

Therefore:

\[ E = IAt = (10^{-2.4})(2.00 \times 10^{-4})(8.50 \times 3600) = 2.44 \times 10^{-2} J \]

d. Evaluate the following statements (T/F/N). As always, explain your reasoning.

(i) The loudness (\( \beta \) value) of a sound can never be less than 0 dB.

**False**: The dB scale was calibrated to human hearing; 0 dB corresponds to the faintest intensity that humans (on average) can hear: \( I_o = 10^{-12} \) W/m^2. But sound can be less intense than that (say, \( I = 10^{-13} \) W/m^2, which would be \( \beta = -10 \) dB)—and indeed, other animals can hear in that range. So the dB scale does include negative values—it’s just that we humans can’t hear such faint sounds.

(ii) The intensity of a 100 dB sound is 10 times more than the intensity of a 10 dB sound.

**False**: Every 10 dB increase is a 10-fold increase in intensity, so 100 db would be \( 10^3 \) (1 billion) times more intense than 10 db.

(iii) A 100 dB sound is twice as loud to you (in your brain) as a 50 dB sound.

**False**: Every 10 dB increase is a 2-fold increase to your brain, so 100 db would be \( 2^5 \) (32) times louder to your brain than 50 db.

(iv) The decibel scale is (physically) unitless. **True**: \( \beta = 10·\log(I/I_o), \) and \( (I/I_o) \) is physically unitless.

(v) The intensity of a sound can never be less than 0 W/m^2.

**True**: The intensity of sound is defined as the mechanical power arriving per area, and mechanical power (mechanical energy per second) arriving is never negative. (If mechanical power were departing from your ear, this would not be sound arriving.)

(vi) If \( \beta_A = 80 \) dB and \( \beta_B = 85 \) dB, then \( I_B/I_A < 3.50. \)

**True**: \( \beta_B - \beta_A = 5 = 10·\log(I_B/I_o) - 10·\log(I_A/I_o) = 10·\log(I_B/I_A) \quad \text{thus:} \quad I_B/I_A = 10^5 = 3.16 \)

(vii) If two mowers (same power and distance away) are operating, then one shuts off, the sound to your ears and brain is then half as loud as before.

**False**: The intensity is cut by half (divided by two), but in order to cut the sound value in your ears and brain by half, you’d have to cut the intensity by 10-fold (divide it by 10).

e. Super Bowl attendees can be quite loud at times. When one football fan (sitting in the stands) screams, suppose that the resulting sound intensity level at midfield is about 80.0 dB. About how many identical fans would need to scream (from the same distance) in order to produce a sound intensity level of at least 110 db at midfield?

Every increase of 10 dB is a 10-fold increase in intensity. So a 30-point increase in \( \beta \) corresponds to a \( 10^3 \)-fold increase in intensity. So **1000 identical fans** would need to be screaming to deliver a 110 dB sound at midfield.

f. A person is standing near a jet airplane with four identical engines. The loudness is bordering on painful: 123 dB. What would the loudness level be if the captain were to shut down all but one engine?

When three out of the four engines are shut down, this cuts the intensity (I) by a factor of 4:

\[ I_f = I/4 \]

If \( \beta_f = 123 \) dB, then:

\[ 10·\log(I/I_o) = 123 \quad \text{So:} \quad I/I_o = 10^{12.3} \quad \text{Or:} \quad I = 10^{12.3}(10^{-12}) = 10^{0.3} \]

Therefore:

\[ I_f = I/4 = (0.25)10^{0.3} \]

And:

\[ \beta_f = 10·\log(I/I_o) = 10·\log[(0.25)10^{0.3}/10^{12} ] = 117 \text{db} \]
3. g. Two sources of sound are located on the $x$-axis, and each emits power uniformly in all directions. There are no reflections. One source is positioned at the origin. The other source is at $x = 186$ m. The source at the origin emits four times as much power as the other source. Find the two positions along the $x$-axis where the two sounds are equal in intensity.

First note: Since one speaker is more powerful than the other, in order for the intensities of their sounds to be equal to you, you will need to stand somewhat nearer to the weaker speaker (B). There are two places that could be: somewhere between the two speakers (but nearer to B); and somewhere to the right of speaker B. (Note that there is no way you could ever hear equal intensities by standing left of speaker A, since it—the more powerful—would always be nearer to you there.)

As it turns out, if you set up the problem to solve for one position (either one), careful algebra—i.e. using both solutions of a square root or quadratic—will give you both solutions. So, suppose you stand somewhere between the speakers, at a distance $X$ from the origin. That means you are standing at a distance $X$ from speaker A and a distance of $(186 - X)$ from speaker B.

If $X$ is the right spot, the intensities are equal: $I_A = I_B$

In other words: $P_A/(4\pi r_A^2) = P_B/(4\pi r_B^2)$

But $P_A = 4P_B$. Therefore: $4P_B/(4\pi r_A^2) = P_B/(4\pi r_B^2)$

And we’ve observed that $r_A = X$, and $r_B = 186 - X$: $4P_B/(4\pi X^2) = P_B/[4\pi(186 - X)^2]$

Simplify, by dividing both sides by $P_B/(4\pi)$: $4/X^2 = 1/(186 - X)^2$

Now, at this point, you have a choice. You can either clear the fractions on both sides—to create and solve a quadratic equation (perfectly logical and correct); or you can do this:

Multiply both sides by $X^2$: $4 = X^2/(186 - X)^2$

Take the square root of both sides: $\pm 2 = X/(186 - X)$

Take each possible solution one at a time: $2 = X/(186 - X)$

Solving this for $X$ gives this result: $X = 372/3 = 124$ m

Take the other possible solution: $-2 = X/(186 - X)$

Solving this for $X$ gives this result: $X = 372$ m

The intensities of the sound from the two speakers will be equal if you stand at distance of either 124 or 372 m from the origin (where speaker A is located). Both of these solutions should make sense: They are locations where you are twice as far from a speaker that is 4 times more powerful. This is exactly the result we would expect from intensities; they are proportional to $1/r^2$. In other words, “to reduce the intensity by a factor of 4, you must stand 2 times as far away.”

3. h. You’re sitting midway between two loudspeakers, A and B, each emitting its maximum power. If you hear speaker A’s sound at 87 dB and speaker B’s sound at 75 dB, how many times more powerful is A than B?

The question being asked is how the powers of the two speakers compare: What is the ratio $P_A/P_B$?

Note: If $\beta_A = 87$ dB, then: $10\log(I_A/I_0) = 87$

And: If $\beta_B = 75$ dB, then: $10\log(I_B/I_0) = 75$

Therefore: $\beta_A - \beta_B = 87 - 75 = 12 = 10\log(I_A/I_0) - 10\log(I_B/I_0)$

Or: $1.2 = \log(I_A/I_0) - \log(I_B/I_0) = \log[(I_A/I_0)/(I_B/I_0)] = \log(I_A/I_B)$

Thus: $10^{1.2} = I_A/I_B = [P_A/(4\pi r_A^2)]/[P_B/(4\pi r_B^2)]$ But $r_A = r_B$ (you’re midway between the speakers)

Thus: $10^{1.2} = I_A/I_B = P_A/P_B = 15.9$
4. a. Evaluate the following statements (T/F/N). As always, explain your reasoning.

(i) If a honking car is traveling at constant speed directly toward a stationary observer, the observed frequency changes as the car gets closer.
   \[ f_O = f_S \frac{1}{1 - \frac{v_s}{v}} \]
   \( f_O \) isn’t changing, neither is \( f_O \).
   \textbf{False}.

(ii) If a sound is getting louder but the source’s emitted power is not changing, the frequency it is emitting is lower than the frequency you’re hearing. (Assume no obstacles, earplugs, etc.)
   The only other explanation for the sound to be getting louder is that you and the source are getting closer together, in which case (see also 4b, below) there would be a Doppler up-shift in frequency: \( f_O > f_S \)
   \textbf{True}.

(iii) If a sound is getting louder but the source’s emitted power is not changing, the frequency you’re hearing is lower than the source frequency. (Assume no obstacles, earplugs, etc.)
   The only other explanation for the sound to be getting louder is that you and the source are getting closer together, in which case (see also 9b, below) there would be a Doppler up-shift in frequency: \( f_O > f_S \)
   \textbf{False}.

(iv) If two cars, A and B, are each emitting the same single sound frequency as A moves away from B (car B is stationary), then driver A hears a higher frequency from B than driver B hears from A.
   \[ f_{O.A} = f_S \frac{1}{1 - \frac{v_O}{v_{sound}}} \]
   \[ f_{O.B} = f_S \frac{1}{1 + \frac{v_O}{v_{sound}}} \]
   \[ f_{beat} = f_{O.B} - f_{O.A} = f_S \left( \frac{1}{1 + \frac{v_O}{v_{sound}}} - \frac{1}{1 - \frac{v_O}{v_{sound}}} \right) = f_S \left( \frac{2v_O}{v_{sound}} \right) \]
   Thus: \( f_{beat}/f_S = \frac{2v_O}{v_{sound}} = \frac{2(9/34)}{1} = 0.0523 \)
   \textbf{False}.

(b. If there is a Doppler shift upward in sound frequency, how is the distance between source and observer changing?
   You cannot tell, without more specific information, how each party is moving, but you do know they are getting closer to each other.

(c. Two speakers are each producing the same single frequency of sound. Speaker A is located at \( x = 0.00 \) m. Speaker B is initially located at \( x = 12.0 \) m, resulting in extreme Destructive Interference at \( x = 6.00 \) m. Then speaker B is re-located to \( x = 15.0 \) m, also resulting in extreme Destructive Interference at \( x = 6.00 \) m. The speed of sound in air here is 344 m/s. Evaluate (T/F/N) the following statement. \textbf{Justify your answer fully with any valid mix of words, drawings and calculations.}

   If you were to move at a steady speed of 9.00 m/s from \( x = 1.00 \) m to \( x = 11.0 \) m, you would hear a beat (amplitude) frequency that is about 5.23% of the value of the actual (pitch) frequency.

   \textbf{True}.

Doppler shifts for stationary source and moving observer:

\[ f_{O,A} = f_S \frac{1}{1 - \frac{v_O}{v_{sound}}} \quad f_{O,B} = f_S \frac{1}{1 + \frac{v_O}{v_{sound}}} \]

\[ f_{beat} = f_{O,B} - f_{O,A} = f_S \left( \frac{1}{1 + \frac{v_O}{v_{sound}}} - \frac{1}{1 - \frac{v_O}{v_{sound}}} \right) = f_S \left( \frac{2v_O}{v_{sound}} \right) \]

Thus:

\[ f_{beat}/f_S = \frac{2v_O}{v_{sound}} = \frac{2(9/34)}{1} = 0.0523 \]
4. c. A stationary siren is producing a sound wave described by $P = 0.4 \cdot \cos[18.5x + 6072t]$, where east is the positive $x$-direction. (The $\pm$ here means that the sound is traveling both westward and eastward from the siren.) The resulting sound is heard as 903 Hz by an observer in a vehicle that is traveling westward (somewhere along the $x$-axis) at constant speed. If the sound also echoes off the vehicle and returns to an observer standing by the siren, what echo frequency does that standing observer hear?

The siren’s sound is given by the wave equation, which tells us a lot about the sound: its frequency and wave length—and therefore its speed. (You cannot assume the speed of sound to be 343 m/s—that’s true only in certain conditions of air temperature.) So, using the given wave equation, we know that $6072 = 2\pi f_r$; or $f_r = 966.3888$ Hz And $18.5 = 2\pi/\lambda$, or $\lambda = 0.33963$ m. Therefore, $v = f_r \cdot \lambda = (966.3888)(0.33963) = 328.216$ m/s.

**Important conclusion:** The siren’s source frequency (966) is higher than the frequency observed at the car (903). Therefore, the car must be moving away from the stationary siren (it’s west of the siren). This is the only way you know where the car is in relation to the siren. Only after noting this can we now find the car’s speed—from the Doppler equation relating the siren’s source frequency to the observed frequency at the car.

The observer is moving away from the source, so we use the − sign in the numerator of the formula (and the denominator is simply 1, because the source speed, $v_s$, is zero): $f_o = f_s(1 - v_o/v)$. Now solve for $v_o$:

$$v_o = v(1 - f_o/f_s) = (328.216)(1 - 903/966.3888) = 21.53 \text{ m/s}$$

(This is quite reasonable for the car’s speed—it’s about 47 mph.)

Now, for the echo, the car becomes the source (for the stationary observer at the siren). Its source frequency is that which it receives and reflects: 903 Hz. And since it is moving away from the observer, the Doppler equation for the echo is: $f_o = f_s[1/(1 + v_o/v)]$ (and the numerator is simply 1, because the observer’s speed, $v_o$, is zero).

Plugging in the numbers, we get:

$$f_o = (903)[1/(1 + 21.53/328.216)] = 847 \text{ Hz}$$

The observer standing by the siren hears an echo frequency of 847 Hz from the car.

d. You’re bicycling at a steady speed alongside a railroad track where a long train is standing at rest. This train has several engines hooked together at the front, and the first and last engines are each emitting identical frequencies of 250 Hz on their horns. When you have passed the first engine but are still approaching the last engine, you hear a beat frequency of 6 Hz. Assuming standard atmospheric conditions, how fast are you moving?

Horn 1 is on the front engine (emitting $f_{1,s} = 250$ Hz). Horn 2 is on the last engine (emitting $f_{2,s} = 250$ Hz). The horns are stationary, but as the observer, you’re moving away from horn 1 but toward horn 2.

You hear (observe) two different frequencies....

You hear from horn 1 (a Doppler down-shift): $f_{1,o} = 250[1 - (v_{you}/343)]$

You hear from horn 2 (a Doppler up-shift): $f_{2,o} = 250[1 + (v_{you}/343)]$

Combined, these tones ($f_{2,o} > f_{1,o}$) produce beats: $f_{beat} = f_{2,o} - f_{1,o} = 6$ Hz

In other words: $250[1 + (v_{you}/343)] - 250[1 - (v_{you}/343)] = 6$

Or:

$$[1 + (v_{you}/343)] - [1 - (v_{you}/343)] = 6/250$$

This simplifies a lot (ready now to solve for $v_{you}$):

$$2v_{you}/343 = 6/250$$

so: $v_{you} = 4.12 \text{ m/s}$

e. You’re bicycling east at a steady speed of 4.00 m/s alongside a railroad track where a train is moving at a steady speed of 5.00 m/s toward the west. This train has several engines hooked together at the front, and the first and last engines are each emitting identical frequencies of 200 Hz on their horns. Assuming standard atmospheric conditions, when you have passed the first engine but are still approaching the last engine, what beat frequency do you hear?

Horn 1 is on the front engine (emitting $f_{1,s} = 200$ Hz). Horn 2 is on the last engine (emitting $f_{2,s} = 200$ Hz). Horn 1 is a source moving away from you, and as the observer, you’re moving away from it. Horn 2 is a source moving toward you, and as the observer, you’re moving toward it.

You hear from horn 1 (a Doppler down-shift): $f_{1,o} = 200[1 - (4/343)]/[1 + (5/343)] = 194.83 \text{ Hz}$

You hear from horn 2 (a Doppler up-shift): $f_{2,o} = 200[1 + (4/343)]/[1 - (5/343)] = 205.32 \text{ Hz}$

Combined, these tones ($f_{2,o} > f_{1,o}$) produce beats: $f_{beat} = f_{2,o} - f_{1,o} = 10.5$ Hz.
4. f. An airhorn is essentially a tube of air that vibrates at certain resonant frequencies, just like many musical instruments. The following questions refer to two airhorns, each 85.0 cm long, operating in the same still air (no winds, same temperature, etc.), where the speed of sound is 340 m/s. Horn 1 is a tube that is open at both ends. Horn 2 is a tube that is open at one end but closed at the other.

(i) Evaluate (T/F/N) the following statement, and explain your reasoning fully. When horn 1 produces a standing sound wave that has 3 pressure antinodes, that sound is one octave higher in pitch (frequency) than when horn 2 produces a standing sound wave with 2 pressure antinodes.

(ii) Suppose that horn 1 is located at the origin (0, 0), horn 2 is at (100, 0), and you’re standing at (75, 0). All coordinates here are given in meters. You are holding a sound meter, and when each horn is sounded by itself, you get these readings: Horn 1: 76.0 dB Horn 2: 92.0 dB Find the ratio of the two horns’ power outputs. That is, calculate $P_1/P_2$. (For this portion, assume no echoes or other complications.)
4. f. (iii) Now suppose you discard horn 2. Then you hold horn 1 and walk at a steady speed of 2 m/s directly toward a tall vertical wall that echoes readily. Calculate the beat frequency you will hear if horn 1 is sounding its fundamental frequency.
5. a. Loudspeaker A is sitting at the origin. Loudspeaker B is sitting at \( x = 10.0 \text{ m} \). Speaker B is 3 times as powerful as speaker A. You are sitting somewhere between the two speakers (on the x-axis), listening as each speaker emits sound at its full power, one at a time (they are not both sounding at once). In your own estimation—judging each sound as you experience it—speaker B sounds twice as loud as from speaker A. Where are you sitting?

The picture:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>(you)</td>
</tr>
<tr>
<td>( P )</td>
<td>( x = ? )</td>
</tr>
<tr>
<td>( x = 0 )</td>
<td>( x = 10 )</td>
</tr>
</tbody>
</table>

If speaker B sounds twice as loud to you as A, then \( I_B = 10I_A \).

That is: \( \frac{P_B}{(4\pi r_B^2)} = \frac{10P_A}{(4\pi r_A^2)} \)

Or: \( 3P_A/[4\pi(10 – r_A)^2] = 10P_A/(4\pi r_A^2) \)

Simplify: \( \frac{3}{(10 – r_A)^2} = \frac{10}{r_A^2} \)

Thus: \( (10 – r_A)/r_A = \sqrt{3/10} \)

Thus: \( r_A = \sqrt{(10 – \sqrt{3}/10)} \) \( = 6.46 \text{ m} \)

You’re sitting at \( x = 6.46 \text{ m} \).

b. Persons A and B are both listening to the same loudspeaker. Person A is sitting somewhere directly north of the speaker; person B is sitting somewhere directly east of the speaker. The sound reaching person A is 4 dB louder than the sound reaching person B. Person A is sitting 22.0 m from person B. How far is person B sitting from the speaker?

If \( \beta_A – \beta_B = 4 \), then: \( \log(I_A/I_B) = 0.4 \).

That is: \( \frac{P}{(4\pi r_A^2)} = (10^{0.4})\frac{P}{(4\pi r_B^2)} \)

But we also know: \( r_A^2 + r_B^2 = 22^2 \)

Thus: \( r_B^2 = (10^{0.4})(484 – r_B^2) \)

Or: \( r_B = \sqrt{(10^{0.4})(484)/(1 + 10^{0.4})} = 18.6 \text{ m} \)

Person B is sitting 18.6 m from the speaker.
5. c. Referring to point X in the diagram below, evaluate the following statements (T/F/N). As always, explain your reasoning.
   (i) \( I_B = 2I_A \)
   (ii) \( \beta_B > (\beta_D + 10 \text{ dB}) \)
   (iii) Sound from C will be twice as loud to your ears (brain) as sound from A.
   (iv) Sound from A is louder than sound from C.
   (v) You hear a higher frequency from B than from D.
   (vi) Speaker C is emitting more power than speaker D.

d. Again referring to point X in the diagram below, assume that all speakers are motionless, and that only the two mentioned in each statement are sounding. Evaluate the following statements (T/F/N). As always, explain your reasoning.
   (i) If \( d = (7.5)\lambda \), then B and C produce Destructive Interference (D.I.).
   (ii) If \( d = (4.0)\lambda \), then B and D produce Destructive Interference (D.I.).
   (iii) If \( d = (0.5)\lambda \), then A and D produce Constructive Interference (C.I.).

Four identical speakers, with current positions as shown, are emitting identical (spherical) sound waves (at the same power and the same single frequency, in phase). The speakers are all moving toward point X, and \( v_A = v_B = v_D > v_C \).
5. e. A loudspeaker is emitting a 2000 Hz tone in a large outdoor amphitheater. There is a standing wave in the back of the space (near some walls). The distance between adjacent muffled spots in that wave is 8.70 cm. But you are not sitting in that wave. You are sitting directly in front of the speaker—though still at some distance from it. The sound takes 0.0731 s to reach you from the speaker. When it does, its loudness is 93.1 dB. Find the power of the speaker.

In any standing wave, the distance between adjacent nodes (the muffled spots) is a half-wavelength.

Thus:  \( \lambda/2 = 0.087 \text{ m} \)

So:  \( \lambda = 0.174 \text{ m} \)

The frequency of this wave is 2000 Hz, so the local speed of sound is:  \( v = f\lambda = 2000(0.174) = 348 \text{ m/s} \)

Now calculate the distance from the loudspeaker to where you’re sitting:  \( d = vt = 348(0.0731) = 25.4388 \text{ m} \)

Knowing that and the loudness (\( \beta \)) at your position, we can calculate the power, \( P \), of the loudspeaker:

\[
\beta = 10 \cdot \log\left(\frac{I}{I_0}\right) = 10 \cdot \log\left\{ \frac{P}{(4\pi d^2 I_0)} \right\} \quad \text{or} \quad \frac{\beta}{10} = \log\left[\frac{P}{(4\pi d^2 I_0)}\right]
\]

\[
10^{\beta/10} = \frac{P}{(4\pi d^2 I_0)} \quad \text{So:} \quad P = (4\pi d^2 I_0) \cdot 10^{\beta/10} = 4\pi(25.4388)^2 \cdot (10^{-12}) \cdot 10^{93.1/10} = 16.6 \text{ W}
\]

f. On a hot afternoon, you’re watching the rehearsal of a marching band on the level stadium field. You are standing 9.48 m from a flute player, who is also standing still, playing a single steady note, which is the 2nd harmonic frequency of the flute (basically a tube that is open at both ends). A trumpet player, standing the same distance away, is trying to play the same note, but his instrument is out of tune, so you hear a beat frequency of 3 Hz. But when the trumpeter marches toward you at a speed of 1.4 m/s, the beats disappear. The speed of sound this day is 350 m/s.

(i) How long is the flute?

(ii) As the trumpeter walks, he reaches points where, momentarily, you can’t hear much from either the flute or the trumpet. One such distance is 7.38 m from you. What is the next distance closer to you?
6. a. A submarine is traveling at speed \( v \) directly toward a stationary underwater sonar buoy. The buoy is emitting a single sound frequency, produced by a tube of length \( L \), which is open at both ends and resonating at its 19th harmonic frequency, with a total power output of \( P \). At the moment when the submarine is located at a certain position \( X \), the sound frequency observed by the submarine is \( f \), and that sound has taken a time interval \( \Delta t \) to travel from the buoy to point \( X \). What is the relative intensity (\( \beta \)) of the sound arriving at point \( X \)?

Assume these are known values: \( v, L, P, f, \Delta t \).

**Equations:**

I. \( f = f_{\text{buoy}}[1 + (v/v_{\text{sound}})] \)

II. \( f_{\text{buoy}} = nv_{\text{sound}}/(2L) \)

III. \( r_f = v_{\text{sound}}(\Delta t) \)

IV. \( I_s = P/(4\pi r_f^2) \)

V. \( \beta_x = 10 \cdot \log(I_s/I_0) \)

**Solving:**

Substitute II into I.

Solve I for \( v_{\text{sound}} \). Substitute the result into III.

Solve III for \( r_f \). Substitute the result into IV.

Solve IV for \( I_s \). Substitute the result into V.

Solve V for \( \beta_x \).

b. A submarine is traveling at constant speed directly toward a stationary underwater sonar buoy. The submarine is emitting a single, steady sound frequency \( f_s \). When the submarine is at point \( A \), a distance \( x \) from the buoy, the sound it emits requires a time \( \Delta t \) to reach the buoy, and the observed frequency of that sound’s echo returning to the sub is \( f_o \). From the moment when the sub is at point \( A \), how much time will elapse before the sub emits sound that will arrive at the buoy 3 dB louder than the sound emitted from point \( A \)?

Assume these are known values: \( f_s, x, \Delta t, f_o \).

**Equations:**

I. \( v_{\text{sound}} = x/\Delta t \)

II. \( f_{o,1} = f_s[1+(v_{\text{sub}}/v_{\text{sound}})]/[1] \)

III. \( f_o = f_{o,1}[1/[1-(v_{\text{sub}}/v_{\text{sound}})] \]

IV. \( 10\log(I_f/I_o) – 10\log(I_i/I_o) = 3 \)

V. \( I_f/I_i = [P/(4\pi r_f^2)]/[P/(4\pi x^2)] \)

VI. \( v_{\text{sub}} = (x – r_f)/\Delta t_{\text{sub}} \)

**Solving:**

Solve I for \( v_{\text{sound}} \). Substitute the result into II and III.

Substitute II into III.

Solve III for \( v_{\text{sub}} \). Substitute the result into VI.

Solve IV for \( I_f/I_i \). Substitute the result into V.

Solve V for \( r_f \). Substitute the result into VI.

Solve VI for \( \Delta t_{\text{sub}} \).
It's a foggy day at the harbor. On the shore is a foghorn made from a tube, \( L \) m long, open at both ends. The foghorn is sounding a steady tone (its fundamental frequency) of \( f \) Hz. A ship is nearing the harbor entrance, moving at some constant velocity. At a certain moment, when the ship is \( D \) km from the horn, the horn’s tone arrives at the ship with a loudness of \( \beta_1 \) dB. Then, \( t \) seconds later, the horn’s tone arrives at the ship with a greater loudness, \( \beta_2 \) dB. Standing on the shore next to the foghorn, what beat frequency will you hear as a result of the foghorn and its echo? (You may assume that the ship is moving either directly toward or away from the foghorn.)

Assume these values are known: \( L, f, D, \beta_1, \beta_2, \) and \( t \).

**Equations:**

I. \( L = n(\lambda/2) \)

II. \( v_{\text{sound}} = f \lambda \)

III. \( \beta_2 - \beta_1 = 10 \cdot \log(I_2/I_0) - 10 \cdot \log(I_1/I_0) \)

IV. \( I_2/I_1 = [P/(4\pi r_2^2)]/[P/(4,000,000\pi D^2)] \)

V. \( v_{\text{ship}} = (D - r_2)/t \)

VI. \( f_o = f_{\text{horn}} [1+(v_{\text{ship}}/v_{\text{sound}})]/[1] \)

VII. \( f_o' = f_s' [1]/[1-(v_{\text{ship}}/v_{\text{sound}})] \)

VIII. \( f_{\text{beats}} = f_o' - f_{\text{horn}} \)

**Solving:**

Solve I for \( \lambda \). Substitute the result into II.

Solve II for \( v_{\text{sound}} \). Substitute the result into VI and VII.

Solve III for \( I_2/I_1 \). Substitute the result into IV.

Solve IV for \( r_2 \). Substitute the result into V.

Solve V for \( v_{\text{ship}} \). Substitute the result into VI.

Solve VI for \( f_o \). Substitute the result into VII.

Solve VII for \( f_o' \). Substitute the result into VIII.

Solve VIII for \( f_{\text{beats}} \).
6. **d.** Two loudspeakers sit at ear level in an empty space, with their aperture centers located at \((0, -d_1)\) for speaker 1 and \((d_2, 0)\) for speaker 2, on the coordinate axes shown in the overhead view here (which is not to scale).

The value of \(d_1\) is not known, but the value of \(d_2\) is known.

Each speaker consists of a cylindrical tube. Speaker 1 is open at both ends; speaker 2 is open at one end, but closed on the other. Speaker 1 is twice as long as speaker 2.

For a certain frequency, \(f\), emitted with the same power \(P\) by each speaker (in phase with each other), you hear almost no sound if you’re listening at point \(A\) (which is directly in front of both speakers). But \(f\) is not the only frequency that behaves like this at point \(A\). There are three lower frequencies that do likewise.

Also: If you were to stand at point \(A\) and listen to each speaker individually (i.e. with the other turned off), then speaker 1 would sound \(L\) times louder to you (to your brain) than speaker 2.

Point \(A\) has the coordinates shown. \(f\) is the 19th harmonic of speaker 1.

**How long is speaker 2?**

**The list of known values:** \(d_2, f, P, x, y, L\).

For this item, use the full seven-step ODAVEST problem-solving procedure. Keep in mind that you’re not being asked to actually solve for the final expression. In fact, you’re not being asked to do any math at all—not even any algebra. Rather, for the Solve step, you are to write a series of succinct instructions on how to solve this problem. Pretend that your instructions will be given to someone who knows math but not physics. And for the Test step, you should consider the situation and predict how the solution would change if the data were different—changing one data value at a time.

**Objective:** Two loudspeakers (1 and 2) are fixed in place, pointing horizontally in an empty space. Their central axes intersect at a known point \(A\).

Speaker 2’s aperture is centered at a known point on the \(x\)-axis.

Each speaker is a hollow, cylindrical tube, vibrating with the same known sound frequency.

When both speakers sound together, the sound at point \(A\) is very faint.

Such faintness at point \(A\) would also occur for three other frequencies that are lower than the frequency being produced by the speakers in this situation.

When each speaker is heard separately by a human ear positioned at point \(A\), speaker 1’s sound is a known factor louder to that ear (and brain) than speaker 2’s.

Speaker 1 is twice as long as speaker 2.

Speaker 1 is vibrating at its 19th harmonic frequency.

**We want to find the length of speaker 2.**

**Data:**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_2)</td>
<td>The distance along the (x)-axis from the origin to the center of speaker 2’s open end.</td>
</tr>
<tr>
<td>(f)</td>
<td>The frequency being emitted by each speaker.</td>
</tr>
<tr>
<td>(P)</td>
<td>The power being output by each speaker.</td>
</tr>
<tr>
<td>(x, y)</td>
<td>The axial coordinates of point (A).</td>
</tr>
<tr>
<td>(L)</td>
<td>The factor by which speaker 1 would sound louder than speaker 2 (in the human brain) if each were heard individually from point (A).</td>
</tr>
</tbody>
</table>
Assumptions: Conditions We assume the air in this space is calm—no breeze or air currents or temperature differences—thus carrying the sound at one uniform speed within the space.

Space We also assume there are no other active sources of sound anywhere near enough to contribute to the sound arriving at point A.

We assume that there are no walls or other objects anywhere near enough to provide any echoes back to point A.

Speakers We assume that each speaker’s sound energy output is uniform enough to model as spherical.

Definitions We assume that the brain response data here refers to the human average—so that the decibel scale can be applied.

We also assume that “you hear almost no sound” signifies completely (perfectly) Destructive Interference.

Visual Rep(s):

![Diagram](image)

Equations:

I. \[ r_2^2 = (|x| + d_2)^2 + y^2 \]

II. \[ |r_2 - r_i| = (m + 1/2)\lambda. \]

III. \[ m = 0, 1, 2, 3, \ldots, \] so from the description given, \( m = 3 \) here.

IV. \[ L = 2^{\log(I_2/I_1)} \] (If \( I_1/I_2 = 10, L = 2 \); if \( I_1/I_2 = 100, L = 4 \); if \( I_1/I_2 = 1000, L = 8 \) ...)

V. \[ l_1 = P/(4\pi r_1^2) \]

VI. \[ l_2 = P/(4\pi r_2^2) \]

VII. \[ l_1 = 19(\lambda/2) \]

VIII. \[ l_1 = 2l_2 \]

Solving: Solve I for \( r_2 \). Substitute that result into II and VI.

Substitute III into II.

Substitute V into IV.

Substitute VI into IV.

Solve IV for \( r_1 \). Substitute that result into II.

Solve II for \( \lambda \). Substitute that result into VII.

Solve VII for \( l_1 \). Substitute that result into VIII.

Solve VIII for \( l_2 \).
Testing: Dimensions: \( l_2 \) should have units of length.

Dependencies: If \( d_2 \) is greater (all other data being unchanged), then \( r_2 \) will increase, so the difference between \( r_2 \) and \( r_1 \) must increase (because \( r_2/r_1 \) must remain the same, since \( L \) is unchanged). This will produce a larger wavelength, \( \lambda \), which means that \( l_2 \) will be greater.

The value of \( f \) was not needed in the problem.

Because the value of \( P \) is the same for both speakers, increasing \( P \) does not affect the resulting value of \( l_2 \).

If \( x \) is greater in magnitude (all other data being unchanged), then \( r_2 \) will increase, so the difference between \( r_2 \) and \( r_1 \) must increase (because \( r_2/r_1 \) must remain the same, since \( L \) is unchanged). This will produce a larger wavelength, \( \lambda \), so \( l_2 \) will be greater.

If \( y \) is greater (all other data being unchanged), then \( r_2 \) will increase, so the difference between \( r_2 \) and \( r_1 \) must increase (because \( r_2/r_1 \) must remain the same, since \( L \) is unchanged). This will produce a larger wavelength, \( \lambda \), which means that \( l_2 \) will be greater.

If \( L \) is greater (all other data being unchanged), then \( r_1 \) will decrease, so the difference between \( r_2 \) and \( r_1 \) must increase (because \( r_2/r_1 \) must remain the same). This will produce a larger wavelength, \( \lambda \), which means that \( l_2 \) will be greater.