The formats (type, length, scope) of these Prep problems have been purposely created to closely parallel those of a typical exam (indeed, these problems have been taken from past exams). To get an idea of how best to approach various problem types (there are three basic types), refer to these sample problems.
1. a. Evaluate the following statements (T/F/N). As always, explain your reasoning.

(i) Light incident from air onto glass at 0° does not enter the glass.  
\textbf{False.} An incidence angle, remember is measured from the \textit{normal} to the glass surface; light striking the glass surface perpendicularly certainly enters the glass.

(ii) If two panes of glass (same material) have different thicknesses, then blue light could pass through the thicker pane in less time than red light passes through the thinner pane.  
\textbf{Not enough information.} Blue light will be transmitted more slowly, yes, so if you were to specify that each case has the light incident normally ($\theta_i = 0^\circ$), then the claim would be false (i.e. the blue light would always “lose the race,” since it’s been handicapped both by speed and distance). But the incidence angles were not specified—and if you aim the (faster) red light at an angle sufficiently oblique so that its path is sufficiently longer than that of the (slower) blue light, red could end up “losing the race.”

(iii) \textbf{White light can refract through glass without separating into colors.}  
\textbf{True.} Send it through a pane of glass (both sides parallel) at an incidence angle of 0°.

(iv) If you’re viewing from underwater, the ceiling of an indoor swimming pool appears higher (farther from you) than it really is.  
\textbf{True.} With the rays entering the water and being bent toward the normal, they appear to be coming from points much higher above. Look at Fig. 23.14a (and take the point source case), in the text book and extrapolate upward—in straight lines—the rays reaching your eye in the water.

(v) \textbf{In the diagram, at least 2 rays are incorrectly drawn.} \hspace{1cm} 
[Diagram: All rays are the same (single). Surfaces between the three regions are parallel. \hspace{1cm} \textit{\ldots} = normal to these surfaces.]

\hspace{1cm} not for positive $n$ values; negative indices of refraction involve physics/math beyond this course). Then compare rays A and B:  
If $n_1 < n_2$, all rays should refract toward the normal (so ray A is correct; B is not). But if $n_1 > n_2$, all rays should refract away from the normal (so ray A is incorrect and ray B is correct).

(vi) In the diagram, if ray B is correctly drawn, then $n_2 < n_1$.  
\textbf{True.} See argument in item (v), above.

b. The light ray shown is traveling initially in the positive $x$-direction. In which direction (expressed as an angle measured from the positive $x$-axis) will it be traveling after refracting through (exiting on the right of) the darker material?  
\hspace{1cm} 
[Diagram: parallel rays (white light). $n_{\text{water}} = 1.33$ (your eye) $n_{\text{water}} < 1.33$]

\hspace{1cm} $n = 1.40$ $70^\circ$ $78^\circ$ $n = 1.40$

\hspace{1cm} \text{red} \hspace{1cm} \text{blue} \hspace{1cm} \text{red} \hspace{1cm} \text{blue}

\hspace{1cm} parallel rays \hspace{1cm} (white light)

\hspace{1cm} \text{(your eye)} \hspace{1cm} \text{blue} \hspace{1cm} \text{red}

\hspace{1cm} \text{red} \hspace{1cm} \text{blue} \hspace{1cm} \text{red} \hspace{1cm} \text{blue}

\hspace{1cm} \text{higher in the sky (and a wider arc—bigger circle).}
2. a. Evaluate the following statements (T/F/N). As always, explain your reasoning.

(i) A critical angle is an angle of incidence.
   True. \( \theta_C \) is the angle of incidence that produces an angle of refraction of 90°.

(ii) For light emerging into air, \( \theta_C \) for water is less than \( \theta_C \) for glass.
   False. \( \theta_C = \sin^{-1}(n_2/n_1) \), where \( n_2 = n_{air} \approx 1 \). Since \( n_{water} \approx 1.33 \), and \( n_{glass} \approx 1.5 \), we have
   \[ \theta_{C,water,air} = \sin^{-1}(1/1.33) = 48.8° \] and \[ \theta_{C,glass,air} = \sin^{-1}(1/1.5) = 41.8° \]

(ii) A diamond is less “sparkly” when immersed in water.
   True. The “sparkle” of a gemstone comes from its ability to temporarily “trap” incident light (send it around a path of total internal reflection) until it encounters a surface at a small enough angle of incidence to escape (that surface being “aimed”—by the prudent cutting of facets in the stone—upward toward the eye). So the smaller the critical angle, \( \theta_C \), the more effective the gem is in redirecting light toward the eye; that is, less light “leaks” out via other faces in the stone.
   Well, dipping a diamond in water increases the critical angle:
   \[ \theta_{C,diamond,air} = \sin^{-1}(1/2.4) = 24.6° \]
   \[ \theta_{C,diamond,water} = \sin^{-1}(1.33/2.4) = 33.7° \]

b. The glass core of an optical fiber has an index of refraction 1.60. The index of refraction of the cladding is 1.48. What is the maximum angle a light ray can make with the wall of the core if it is to remain inside the fiber?
   We want total internal reflection at the core/cladding interface: \( \theta_{C,core,cladding} = \sin^{-1}(1.48/1.60) = 67.7° \)
   That’s the minimum incidence angle—measured from the normal to the core wall. So the maximum angle the ray can make, measured from the core wall itself is \( 90° - \theta_{C,core,cladding} = 22.3° \)

c. A solid glass cube, of edge length 10.0 mm and index of refraction 1.75, has a small, dark spot at its exact center. Find the minimum radius of black paper circles that could be pasted at the center of each cube face to prevent the center spot from being seen, no matter what the direction of viewing.

(before pasting paper circles)             (after pasting paper circles)
3. a. Evaluate the following statements (T/F/N). As always, explain your reasoning.
   
   (i) A diverging lens always produces a positive magnification.
   **True.** For any negative focal length (that’s a diverging lens) and any positive object distance, the thin lens equation will always produce a negative image distance—and thus a positive \( m \) value (since \( m = -d_i/d_o \)). The image will not be inverted with respect to the object.
   
   (ii) For thin lenses, any real image is inverted.
   **True.** Any real image has a positive image distance—producing a negative \( m \) value (since \( m = -d_i/d_o \)).
   
   (iii) When a lens has a positive focal length and an object is placed at a distance equal to twice that focal length, its image will be virtual.
   **False.** Using the thin lens equation, if \( d_o = 2f \), then \( d_i \) will be positive (also \( 2f \)), signaling a real image.
   
   (iv) For a converging lens, the focal length for blue light is slightly shorter than the focal length for red light.
   **True.** The index of refraction in any medium will be slightly higher for a higher frequency, which results in more pronounced refractive angles—thus, shorter focal lengths—for those higher frequencies. Blue light has a higher frequency than red light.
   
   (v) For a thin lens, any virtual image is reduced (smaller than the object).
   **False.** For a converging lens, if \( d_o < f \), then the image will be virtual but enlarged.
   
   (vi) The magnification produced by a diverging lens is always less than 1.
   **True.** The image will always be virtual but non-inverted and reduced (thus \( 0 < m < 1 \)).
   
   (vii) A diverging lens produces only virtual images.
   **True.** The image will always be virtual but non-inverted and reduced (thus \( 0 < m < 1 \)).
   
   (viii) A converging lens can produce an upright, enlarged image.
   **True.** For a converging lens, if \( d_o < f \), then the image will be upright and enlarged.
   
   (ix) A lens that causes parallel light rays to all pass through one common focal point is thicker in its middle than at its edges.
   **Not enough information.** This description (of a converging lens) is true so long as the medium surrounding the lens has an index of refraction less than the material forming the lens, but if the surrounding medium has a higher index, then “fatter-in-the-middle” would producing diverging, instead.

b. Which image distances are possible for a single converging lens, that has a focal length \( f \), creating an image from a single object? As always, explain your reasoning fully.

A comment here: The question stipulates a single converging lens creating an image from a single object—so only positive object distances are possible. If we allow for this lens to create its image by using another lens’ image as its object, then that object distance could indeed be negative (as in part Vc of Lab 6), so the lens could produce any of these three image distances.

First, solve the thin lens equation \( (1/f) = 1/d_o + 1/d_i \) for \( d_o \):

\[ d_o = (f)(d_i)/(d_i-f) \]

Now use this to test each \( d_i \) value below. If the result is a positive \( d_o \), then the \( d_i \) is possible.

(i) \(-2f\) \quad \( d_o = (f)(-2f)/(-2f-f) = (2/3)f \) 
   For a positive \( f \), this is a positive \( d_i \): **Possible.**

(ii) \((1/3)f\) \quad \( d_o = (f)(f/3)/(f/3-f) = -(1/2)f \) 
   For a positive \( f \), this is a negative \( d_o \): **Not possible.**

(iii) \((5/4)f\) \quad \( d_o = (f)(5f/4)/(5f/4-f) = (5)f \) 
   For a positive \( f \), this is a positive \( d_o \): **Possible.**

c. Evaluate the following statement (T/F/N). As always, explain your reasoning.

\[ \text{If } |f| = |f_2| = f \text{ in the situation shown, then the overall lateral (or absolute) magnification, } m, \text{ is } -1.00. \]

**False.**

Lens 1: \( 1/(-f) = 1/(2f) + 1/d_{o,1} \) 
Or: \(-3/(2f) = 1/d_{o,1} \) 
Thus: \( d_{o,1} = -2f/3 \) 
And: \( m_1 = -(2f/3)/(2f) = 1/3 \)

Lens 2: \( 1/f = 1/(5f/3) + 1/d_{o,2} \) 
Or: \( 2/(5f) = 1/d_{o,2} \) 
Thus: \( d_{o,2} = 5f/2 \) 
And: \( m_2 = -(5f/2)/(5f/3) = -3/2 \)

So: \( m_{\text{total}} = m_1m_2 = (1/3)(-3/2) = -1/2 \)
4. In the drawing, the lenses have focal lengths of $f_A = 4$ cm, $f_B = 12$ cm, and $f_C = -8$ cm. The object is located at $x = 0.00$.

a. Find the total magnification ($m$) of this lens system. That is, find the ratio $h_{i,\text{final}}/h_o$.

b. What is the $x$-position of the final image?

Lens A: \[ \frac{1}{4} = \frac{1}{8} + \frac{1}{d_i} \]

The image of lens A (i.e. the object of lens B) is located 8 cm to the right of lens A (4 cm to the left of lens B).

\[ d_i = 8 \quad m_A = -\left(\frac{8}{8}\right) = -1 \]

Lens B: \[ \frac{1}{12} = \frac{1}{4} + \frac{1}{d_i} \]

The image of lens B (i.e. the object of lens C) is located 6 cm to the left of lens B (16 cm to the left of lens C).

\[ d_i = -6 \quad m_B = -\left(\frac{-6}{4}\right) = 1.5 \]

Lens C: \[ -\frac{1}{8} = \frac{1}{16} + \frac{1}{d_i} \]

\[ d_i = -5.33 \quad m_C = -\left(\frac{-5.33}{16}\right) = 0.333 \]

a. \[ m_T = m_A \cdot m_B \cdot m_C = (-1)(1.5)(0.333) = -0.500 \]

b. The final image position is 5.33 cm to the left of lens C. That’s $x = (30 - 5.33) = 24.7$ cm

c. The object shown here is located inside the focal length of lens A. Will the resulting final image be virtual or real?

Upright or inverted?

Since the $d < f$ for the converging lens (A), the image of lens A (which becomes the object of lens B) is virtual—located somewhere to the left of the object—and it is upright. That object position (as with every object position for a diverging lens) will then produce a virtual, upright image (smaller and closer to lens B).

d. The two lenses shown here are separated by a distance equal to the focal length of lens B. The object distance is less than the lens separation distance. Will the resulting final image be virtual or real? Upright or inverted?

The image of lens A (i.e. the object of lens B) is located somewhere between the object and lens A. This is outside the focal point of lens B, so lens B will produce a real, inverted image (somewhere to the right of lens B).
4. e. Two lenses are placed 60.0 cm apart. When an object 4.00 cm tall is placed 20.0 cm in front of the first lens, the final image (from the second lens) is located halfway between the two lenses. If the magnification, $m_1$, of the first lens is –2.50, find the final image’s height and also its orientation (upright or inverted) with respect to the original object.

f. An object is 18 cm in front of a diverging lens that has a focal length of magnitude 12 cm. Calculate how far in front of the lens the object should be placed so that the size of its image is reduced by a factor of 2.0. We want $m_B = m_A/2$ (same object height but half the image height).

First, notice: $1/f = 1/d_o + 1/d_i$  
So: $d_o/f = 1 + d_o/d_i = 1 - 1/m$  
Thus: $m = f / (f - d_o)$

Find $m_A$: $m_A = f / (f - d_{oA}) = -12/(-12 - 18) = 0.4$

So we want: $m_B = m_A/2 = 0.2$

Thus: $0.2 = f / (f - d_o)$  
Or: $0.2 = f / (f - d_o)$

Thus: $d_o = f - (f/0.2) = -12 -(-12/2) = 48.0 \text{ cm}$
5. Evaluate the following statements (T/F/N). As always, explain your reasoning.

a. A near-sighted person’s near point is farther than is considered “normal.”
   False. A near-sighted person’s near point is “normal.” His/her far point is nearer than is considered “normal.”

b. A near-sighted person’s far point is nearer than is considered “normal.”
   True. This is what needs correcting. The person would like to be able to view objects at the normal far viewing limit ($d_{o,normal} = \infty$).

c. A far-sighted person’s far point is farther than is considered “normal.”
   False. A far-sighted person’s far point is “normal.” His/her near point is farther than is considered “normal.”

d. A diverging lens can correct nearsightedness.
   True. The person would like to be able to view objects at the normal far viewing limit ($d_{o,normal} = \infty$), and a diverging lens does exactly what’s needed: Produce a nearer, upright image from a farther upright object.

e. If you have both a contact lens and eyeglasses to correct your nearsightedness (wearing just one at a time), the contact lens (worn on the eye) must have a longer focal length than the eyeglass (worn 2 cm from the eye).
   True. Compare the lens equation for each case ($d_{o,normal} = \infty$).
   
   - Contact lens: $1/f = 1/\infty + 1/-(FP)$
     Thus: $f = -FP$
   - Eyeglasses: $1/f = 1/\infty + 1/-(FP - 2)$
     Thus: $f = -(FP - 2)$
6. a. You wear contact lenses to correct your vision. Your eyes are identical, and the lens for each eye has a refractive power of 2.75 diopters. Find the near point and far point for your eyes.

A positive refractive power (diopters) indicates a positive $f$ length, which is a converging lens—used to correct far-sightedness. A far-sighted person has a normal far point: $FP = \infty$

However, his/her near point (NP) is farther than is considered normal (i.e. $NP > 25$ cm).
And the lens needed to correct this (worn next to the eye) must satisfy $1/f (= RP) = 1/0.25 + 1/-(NP)$

Solving for $NP$: $NP = 0.800$ m (or 80.0 cm)

b. You wear contact lenses to correct your vision. Your eyes are identical, and the lens for each eye has a refractive power of $-2.00$ diopters. Find the near point and far point for your eyes.

A negative refractive power (diopters) indicates a negative $f$ length, which is a diverging lens—used to correct near-sightedness. A near-sighted person has a normal near point: $NP = 0.250$ m (or 25.0 cm)

However, his/her far point (FP) is nearer than is considered normal (i.e. $FP < \infty$).
And the lens needed to correct this (worn next to the eye) must satisfy $1/f (= RP) = 1/\infty + 1/-(FP)$

Solving for $FP$: $FP = 0.500$ m (or 50.0 cm)

c. If your far point is 80.0 cm, but you use a contact lens to correct this, what image distance will you view for an object placed 45 cm from your eye?

Your far point (FP) is nearer than is considered normal (i.e. $FP < \infty$).
And the lens needed to correct this (worn next to the eye) must satisfy $1/f = 1/\infty + 1/-(FP)$

Solving for $f$: $f = -0.800$ m (or $-80.0$ cm)

Then: $1/-80 = 1/45 + 1/d_i$

Solving for $d_i$: $d_i = -28.8$ cm

d. After accidentally breaking his current eyeglasses, a certain professor must use an older pair (with an out-of-date correction) until new glasses are made. The refractive power of the old glasses is 1.660 diopters, and when wearing them (about 2.00 cm from his eyes), he must hold a newspaper at least 42.0 cm from his eyes in order to read it. What is his near point? (A “normal” near point is considered to be 25.0 cm.)

Important points for this problem: First, we know that the image must be virtual (to the left of the lens—so $d_i$ will be negative in the thin lens equation) and upright (not inverted), since his eye needs to view it in the correct orientation but at some distance longer than the 42 cm to the object (otherwise the lens would be unnecessary). We can use the thin lens equation to find $d_i$ if we can first determine the focal length of the lens:

$$f = 1/(r.p.) = 1/(1.66) = 0.60241$ m = 60.241 cm.$$

Now use the thin lens equation: $1/d_o + 1/d_i = 1/f$

Re-arrange to solve for $d_i$: $d_i = (fd_o)/(d_o - f)$

The numbers: $d_i = (60.241)(40)/(40 - 60.241) = -119$ cm

Since the distance from the image is $2$ cm farther to his eye than to the lens, his near point, $NP$, must be $121$ cm. ($NP$ is an absolute value; it carries no sign.)
If a person’s eyes need corrective lenses, they seldom both need exactly the same correction. The type of correction is often the same, of course—but not always. Suppose a person wears eyeglasses (2.00 cm from each eye) to properly correct his vision, as follows: The left lens has a refractive power of 2.00 diopters; the right lens has a refractive power of –1.25 diopters. Now suppose that he removes his eyeglasses to use a simple handheld magnifier of focal length 3.50 cm. He is viewing a tiny plant specimen that is 0.500 mm long.

a. Calculate the maximum angular magnification he can achieve by (properly) using the magnifier with his left eye.
b. Calculate the maximum angular magnification he can achieve by (properly) using the magnifier with his right eye.
c. Approximately what maximum viewing angle (\(\theta'\)), in radians, can he achieve by properly using the magnifier with his left eye?
d. Approximately what maximum viewing angle (\(\theta'\)), in radians, can he achieve by properly using the magnifier with his right eye?
e. Looking at all of the above results (a-d), which eye should he use with the magnifier for the best detailed view of the specimen? Fully explain your decision.
f. How far from the specimen must he hold the magnifier properly for (i.e. next to) his near-sighted eye in order to get the most comfortable (i.e. the most distant-viewing) angular magnification?

a. First, find the near point of this far-sighted eye: \(1/f_{\text{eyeglass}} = 1/d_{\text{e, standard}} + 1/[-(NP - .02)]\)
\[2.00 = 1/0.23 + 1/[-(NP - .02)]\]
Thus: \(NP = 0.4459\) m
Then for a properly used magnifier: \(M_{\text{max}} = NP/f + 1\)
\[= .4459/.0350 + 1 = 13.7\]
b. We may assume that the near point of this near-sighted eye is “standard:” \(NP = 0.25\) m
Then for a properly used magnifier: \(M_{\text{max}} = NP/f + 1\)
\[= .25/.0350 + 1 = 8.14\]
c. By definition, \(\theta' = M\theta\), where \(\theta\) is the eye’s best unaided view of the object.
In this case, that angle is: \(\theta \approx h_o/NP = .0005/0.4459\)
Thus: \(\theta' = M\theta \approx 13.74(.0005/0.4459) = 0.0154\) rad

d. Again, by definition, \(\theta' = M\theta\), where \(\theta\) is the eye’s best unaided view of the object.
In this case, that angle is: \(\theta \approx h_o/NP = .0005/0.25\)
Thus: \(\theta' = M\theta \approx 8.143(.0005/0.25) = 0.0163\) rad

e. If we assume that the eye depths (distances from lens to retina) are equal; or that the near-sighted (right) eye depth is greater (and one or the other of these assumptions seems quite likely), then: We would choose the right eye, because the larger \(\theta'\) it achieves would produce the larger image height on the retina.

(Thus: The angular magnification number, \(M\), doesn’t necessarily tell you which is the better angular view. It tells you which is the most improved angular view. The reason the far-sighted eye’s \(M\) value is bigger is because it’s starting with an inferior unaided view. A greater \(M\) value guarantees the better view only if the two unaided views are equal.)
f. The farthest away the image can be from his eye is its far point, so find it:
\[1/f_{\text{eyeglass}} = 1/d_{\text{e, standard}} + 1/[-(FP - .02)]\]
\[-1.25 = 1/\infty + 1/[-(FP - .02)]\]
Thus: \(FP = 0.820\) m
Then for a properly used magnifier: \(1/f = 1/d_o + 1/[-(FP)]\)
Or: \(1/0.35) = 1/d_o + 1/(-0.820)\)
Solving for \(d_o\):
\(d_o = 0.0336\) m \((or: 3.36\) cm)
8. a. Evaluate the following statements (T/F/N). As always, explain your reasoning.

   (i) If a lens can be used as a magnifier, its refractive power is positive.  
       **True.** A magnifier is a converging lens; its $f$ is positive (and $R.P. = 1/f$).
   
   (ii) If a lens can be used as a magnifier, then no matter how it's used, it always produces upright images.  
       **False.** A converging lens will produce inverted images whenever the object is placed beyond the focal point.
   
   (iii) If a lens can be used as a magnifier, it is the sort of lens used as the eyepiece in a telescope or microscope.  
       **True.** Indeed, this is exactly the function and design of the eyepieces of those instruments: they are simply (very) powerful magnifiers.
   
   (iv) The focal length of a magnifier can never be less than 1 cm.  
       **False.** The focal lengths of many microscope eyepieces, for example, are just a few millimeters.
   
   (v) When used properly, a magnifier always produces positive magnification ($m > 0$).  
       **True.** When the object is placed at or inside the focal point, the image is non-inverted (positive $m$).
   
   (vi) To use a magnifier conventionally, you must locate your eye right behind the lens and the object at the focal point of the lens.  
       **True.** This results in “comfortable viewing,” where the image being placed “way out on the horizon (essentially at $-\infty$).
   
   (vii) In order for a magnifier to produce maximum angular magnification, the object must be located at its focal point.  
       **False.** That would result in comfortable viewing (see item vi, above), but not maximized viewing. For the latter, you need to locate the object a small distance inside the focal point.
   
   (viii) A glass magnifier’s focal length in air is shorter than in water.  
       **True.** The focal length results from the severity of the refractive “bending” of the light coming through the lens. That bending (change of angle—computed by Snell’s Law) is greater for a greater contrast between the the index of refraction of the lens and its surroundings. When you immerse a glass lens ($n \approx 1.50$) in water ($n \approx 1.33$), that contrast is less than for glass in air ($n \approx 1.00$). The bending is therefore greater in air, so the focal length is shorter.

b. Suppose you’re unable to visually distinguish color, but you have two different lasers (red and green), mounted in parallel, side-by-side, aimed at a wall. Explain how a magnifying glass (not necessarily used conventionally) could help you find out which laser is which.

   Place the magnifying glass in front of each lens, in turn, with the optical axis of the lens horizontal (and normal to the wall), and so that the parallel rays of the laser strike the same level on the upper portion of the lens. Then the spot that forms lower on the wall is the green laser; its path is “bent” more severely than the red laser.

c. A student wears contact lenses (2.25 diopters for each eye).

   (i) What absolute magnification ($m$) do these lenses produce when she holds a newspaper at 35 cm from her eye?

   (ii) What angular magnification ($M$) do these lenses produce when she holds a newspaper at 35 cm from her eye?

   (i) The focal length: $f$ (meters) = $1/R.P.$ = $1/2.25$ = 0.444 m

   The image distance: $1/0.444 = 1/0.35 + 1/d_i$

   Thus: $d_i = -1.647$

   The magnification: $m = -(d/d_o) = -(-1.647/0.35) = 4.71$

   (ii) The unaided view: *Could the student even view the newspaper at 35 cm with an unaided eye?*

       Find her near point: $2.25 = 1/25 + 1/(-NP)$

       Her near point os 0.5714 m. *That’s the best she can do.* So: $\theta \approx h_o/0.5714$

       The “aided” view: $\theta' \approx h_i/1.647 = (mh_o)/1.647$

       The angular magnification: $M = \theta'/\theta = [(mh_o)/1.647]/(h_o/0.5714) = [(4.71h_o)/1.647]/(h_o/0.5714) = 1.63$
8. If your eye uses a contact lens of –1.25 diopters to properly correct your vision, and you’re wearing that lens to view an object from a distance of 50.0 cm, …

(i) What is the absolute magnification \((m)\) achieved in this situation?

(ii) What is the angular magnification \((M)\) achieved in this situation?

(i) Find the focal length of this lens: \(f = 1/\text{R.P.} = 1/(-1.25) = -0.800 \text{ m} = -80.0\) 
Now locate the image for an object distance of 50 cm: \(1/(-80) = 1/50 + 1/d_i\) 
Solving for \(d_i\): \(d_i = -30.769 \text{ cm}\) 
Therefore: \(m = -(d_i/d_o) = -(-30.769/50) = 0.615(38)\)

(ii) You’re told to view the object from 50 cm, so your unaided viewing angle is given by \(\theta \approx h_o/50\). 
When wearing the lens—located next to your eye—your viewing distance is the same as \(d_i\) (except for the sign). Therefore, your viewing angle is given (approximately) by \(\theta' \approx h/[d_i]\). 
But \(h_i = m(h_o)\). Substituting: \(\theta' \approx m(h_o)/|d_i|\) 
Therefore: \(M = \theta'/\theta = [m(h_o)/|d_i|]/(h_o/50) = 50m/|d_i| = 50(0.61538)/30.769 = 1.00\)

e. If you have normal vision and properly use a magnifying glass with a refractive power of 12.5 diopters, how far should you hold it from the object for maximum angular magnification?

What if your near point is 40.0 cm?

If you have normal vision, your near point (NP) is 25 cm. And for the maximum viewing angle, you should view the image at your near point, 25 cm from your eye, which (since you hold the magnifier right next to your eye) means that the desired image distance is –25 cm. 
Thus: \(1/d_o + 1/(-.25) = 1/f = 12.5\) Solve for \(d_o\): \(d_o = 0.0606 \text{ m}\) (or 6.06 cm)

Repeat for \(NP = 0.40 \text{ m}\): \(1/d_o + 1/(-.40) = 1/f = 12.5\) \(d_o = 0.0667 \text{ m}\) (or 6.67 cm)

f. Compared to your best unaided view of an object, if you hold a magnifier (\(f = 8.00 \text{ cm}\)) next to your eye (\(NP = 72.0 \text{ cm}\)), what angular magnification \((M)\) do you get by viewing the object 7.50 cm from your eye?

You can’t view any object any closer than your near point (NP), 72 cm, so your best unaided viewing angle is given (approximately) by: \(\theta \approx h_o/72\) 
Now locate the image produced by the magnifier: \(1/f = 1/d_o + 1/d_i\) 
That is: \(1/8.00 = 1/7.50 + 1/d_i\) Solve for \(d_i\): \(d_i = -120 \text{ cm}\) 
Find the absolute magnification produced by the magnifier: \(m = -(d_i/d_o) = -(120/7.50) = 16.0\) 
Since the magnifier is located next to your eye, your viewing distance of its image is the same as \(d_i\) (except for the sign). Therefore, your viewing angle is given (approximately) by \(\theta' \approx h/[d_i]\). 
But: \(h_i = m(h_o) = 16h_o\) Substituting: \(\theta' \approx 16h_o/[d_i] \approx 16h_o/120 = 0.13333h_o\) 
Therefore: \(M = \theta'/\theta = 0.13333h_o/(h_o/72) = 0.13333(72) = 9.60\)

g. You and a colleague are out looking for rare plants. Your colleague is nearsighted, but she wears eyeglasses to correct it. Her correction is -3.75 diopters for each lens. You have normal vision—no correction. You’re trying to use a magnifying glass to view a small plant growing in rock crevice, but you can’t get the large magnifying lens (\(f = 10.0 \text{ cm}\)) close enough to the plant to use it as intended. Your colleague removes one lens from her eyeglasses and is able to place it so that your magnifier offers normal comfortable viewing for you. If the two lenses are 2.50 cm apart, what is the distance from the magnifying glass to the plant?
8. **Suppose you wear eyeglasses with identical 1.25 D (Diopter) lenses to properly correct your vision. And**
**suppose that you dismantle those eyeglasses and place the lenses (A and B) as shown here, in order to**
**enhance your viewing of an object that is 0.1 cm tall. Where, measuring from lens B, must you place your**
**eye so that lenses A and B, acting together, offer you an angular magnification (M) of approximately 4.00?**

First, find the focal length of the lenses: \( 1.25 \text{ D} = \frac{1}{f} \)

So: \( f = 0.800 \text{ m} = 80.0 \text{ cm} \)

Next, find the location of image A:

\[
\frac{1}{f} = \frac{1}{d_{oA}} + \frac{1}{d_{iA}}
\]

Solve for \( d_{iA} \):

\[
d_{iA} = \frac{fd_{oA} - f}{d_{oA}} = \frac{(80)(112)/(112 - 80)}{80} = 280 \text{ cm}
\]

Find \( m_A \):

\[
m_A = -\frac{d_{iA}}{d_{oA}} = -\frac{280}{112} = -2.50
\]

Image A forms 280 cm to the right of lens A. That’s 120 cm to the left of lens B. So \( d_{oB} = 120 \text{ cm} \).

Next, find the location of image B:

\[
\frac{1}{f} = \frac{1}{d_{oB}} + \frac{1}{d_{iB}}
\]

Solve for \( d_{iB} \):

\[
d_{iB} = \frac{fd_{oB} - f}{d_{oB}} = \frac{(80)(120)/(120 - 80)}{80} = 240 \text{ cm}
\]

Find \( m_B \):

\[
m_B = -\frac{d_{iB}}{d_{oB}} = -\frac{240}{120} = -2.000
\]

Find \( m_T \), the total magnification:

\[
m_T = m_A m_B = (-2.5)(-2.000) = 5.000
\]

Thus, the final image height is given by

\[
h_i = 5.000(h_o) = 5.000(0.1) = 0.5000 \text{ cm}
\]

Now, we want an angular magnification of about 4.00. That is, \( M = \theta'/\theta = 4.00 \), where \( \theta \) represents the best unaided view of the original object. With no restrictions on how you can view the object with just your own eye, the best view is when you hold the object at your near point.

**But you are far-sighted. Your near point greater than the standard 25 cm.** To find your near point, note that your eyeglass lens is designed to be worn about 2 cm from your eye, where it must accept light from an object being held at the standard 25 cm from your eye (so, 23 cm from the lens); and then it must deliver the (virtual) image to a distance NP cm from your eye (so, NP – 2 cm from the lens).

So the lens was designed like this:

\[
\frac{1}{80} = \frac{1}{23} + 1 - (NP - 2)
\]

Solve that for NP:

\[
NP = [(80)(23)/(80 – 23)] + 2 = 34.28 \text{ cm}
\]

So your best unaided view of the object is \( \theta \approx h_i/NP = (0.1)/34.28 = 2.9171 \times 10^{-3} \text{ rad} \)

Your aided view must be 4 times that: \( \theta' = 4(2.9171 \times 10^{-3}) = 1.1668 \times 10^{-2} \text{ rad} \)

And that aided view must be \( \theta' \approx h_i/d_{\text{view}} \).

Therefore, your viewing distance must be:

\[
d_{\text{view}} \approx h_i/\theta' = 0.5000/(1.1668 \times 10^{-2}) = 42.9 \text{ cm}
\]

That is, your eye must be located 240 + 42.9 = 283 cm to the right of lens B.
9. a. Evaluate the following statements (T/F/N) for a properly operating, properly adjusted, conventional (2-lens) barrel microscope. As always, explain your reasoning.

(i) The image distance of the first lens is approximately equal to the object distance of the second lens.  
   **False.** The image distance of the first lens (the objective lens) consumes most of the barrel length; the object distance of the second lens (the eyepiece) is very short.

(ii) The microscope inverts the image for the viewer.  
   **True.** The objective lens creates a real image, which is inverted; then the eyepiece creates a virtual image, which does not re-invert the first image.

(iii) The image of the eyepiece is located at the focal point of the objective lens.  
   **False.** The image of the objective lens is located at the focal point of the eyepiece.

(iv) If \( L \) is the distance between the two lenses, and all lengths are expressed in meters, the refractive power of the objective lens could be correctly written as \( 1/(L - d_{o, eyepiece}) + 1/d_{o, objective} \).
   **True.** The thin-lens equation, applied to the objective lens:
   \[
   \frac{1}{f_{objective}} = \frac{1}{d_{o, objective}} + \frac{1}{d_{i, objective}}
   \]
   But if the lengths are expressed in meters, then:
   \[
   \frac{1}{f_{objective}} = R.P._{objective}
   \]
   And in a properly adjusted microscope: \( d_{i, obj} = L - f_e \) and \( f_e = d_{o, eyepiece} \)
   Therefore, in a properly adjusted microscope:
   \[
   R.P._{objective} = 1/d_{o, objective} + 1/(L - d_{o, eyepiece})
   \]

b. For a properly operating, properly adjusted, conventional (2-lens) barrel microscope...

(i) Where should the object be placed? Just **outside** the focal point.

(ii) Where must the objective lens place its image? At the focal point of the eyepiece.

c. To read, you need to wear eyeglasses (2 cm from your eye) to correct your farsightedness (same for each eye). However, you remove those glasses when using your standard, two-lens microscope. The focal length of the objective lens of that microscope is 0.500 cm, and the distance between the two lenses is 16.5 cm. If you achieve a comfortable angular magnification of \(-3840\) when you place the object 0.516 cm from the objective lens, what is the correction (in diopters) of your reading glasses?

First, locate the image formed by the objective lens:
\[
\frac{1}{f_o} = \frac{1}{d_{o, obj}} + \frac{1}{d_{i, obj}}
\]
That is:
\[
1/0.500 = 1/0.516 + 1/d_{i, obj}
\]
Thus: \( d_{i, obj} = 16.125 \) cm

But because the image of the objective lens is located at the focal point of the eyepiece, the distance \( L \), between the two lenses is the sum of \( d_{i, obj} \) and \( f_e \):
\[
L = d_{i, obj} + f_e
\]
Solving for \( f_e \): \( f_e = L - d_{i, obj} = 16.5 - 16.125 = 0.375 \) cm

Then we know: \( M = -(NP)(L - f_e)/(f_o \cdot f_e) \)
Solving for \( NP \): \( NP = -M(f_o \cdot f_e)/(L - f_e) = -(-3840)(0.500)(0.375)/(16.5 - 0.375) = 44.651 \) cm

Now find the eyeglass focal length, \( f \), that will correct that near point:
\[
\frac{1}{f_{glasses}} = \frac{1}{d_{o, glasses}} + \frac{1}{d_{i, glasses}}
\]
That is:
\[
\frac{1}{f_{glasses}} = 1/(25-2) + 1/(-NP - 2)
\]
Thus: \( f_{glasses} = 49.92 \) cm = 0.4992 m

Therefore: \( RP = 1/f_{glasses} (m) = 1/(0.4992) = 2.00 \text{ diopters} \)
10. a. Evaluate the following statements (T/F/n). As always, fully explain your reasoning.

(i) The focal point of a telescope’s objective lens is nearer to the eyepiece than to the objective lens. **True.** The image distance of the first lens (the objective lens) is long—it consumes most of the barrel length—and that image distance is also the focal length of the objective lens; it forms its image right at its own focal point (because it is receiving essentially parallel rays arriving from a very distant object). So the two focal points coincide (see also items b and c below), but then the distance from that common point to the second lens (the eyepiece) is very short, since the eyepiece is a very powerful (very short-focal-length) magnifier.

(ii) For a properly adjusted, standard 2-lens telescope (where \( L \) is the distance between the two lenses), the angular magnification could be correctly written as \(-d_{\text{eyepiece}}/d_{\text{objective}}\). **False.** In a properly adjusted telescope: \( M = -f_o/f_e \)  
But since the rays from the distant object are essentially parallel, the objective lens lands its image right at its own focal point (which is also the focal point of the eyepiece, of course): \( f_o = d_{\text{objective}} \)  
Therefore, in a properly adjusted telescope: \( M = -(d_{\text{objective}})/d_{\text{eyepiece}} \)

b. For a conventional telescope, where should the focal point of the objective lens be located? 
At the focal point of the eyepiece.

c. A two-lens, refracting telescope has an angular magnification of \(-83.0\). The length of the barrel (distance between lenses) is 1.50 m. Find the focal lengths of the two lenses.

The key point to realize about a telescope is that the focal points of the two lenses coincide; they’re located at the same point. This is because the parallel light from the very distant object will form its image right at the focal point of the objective lens; and that location needs to be very nearly (just inside) the focal point of the eyepiece. Hence the co-location:

\[
\text{(very distant object)} \quad \left\arrow{L = 150 \text{ cm}} \right. \quad \left\arrow{f_o = ??} \right. \quad \left\arrow{f_e = ??} \right. \quad \downarrow_{\text{image}_o = \text{object}_e}
\]

Thus: \( L \approx f_o + f_e \) 
Also, we know that for a telescope of this configuration: \( M \approx -f_o/f_e \)

Since we know both \( L \) and \( M \) (but neither \( f_o \) nor \( f_e \)), this is a simple case of two simultaneous equations with two unknowns, which we solve for \( f_e \):

\[ f_e = L/(1 - M) \]

The numbers: \( f_e = 150/[1 - (-83.0)] = 1.79 \text{ cm} \)

And therefore: \( f_o = L - f_e = 150 - 1.79 = 148 \text{ cm} \)

The focal length of the objective is 148 cm. The focal length of the eyepiece is 1.79 cm.
10. d. A standard, two-lens telescope is to be modified with a third converging lens (the “inverter” lens—placed somewhere between the objective lens and the eyepiece) so that the comfortable angular magnification ($M$) of the telescope is changed from $-120$ to $+120$. If the inverter lens is identical to the eyepiece, and the length of the modified telescope (measured from objective lens to eyepiece) is $96.0$ cm, find the focal length of the objective lens.

The angular magnification, $M$, of the un-modified scope is given by: 

$$M = -(f_o/f_e)$$

Thus: 

$$-(f_o/f_e) = -120 \text{ (or: } f_o/f_e = 120)$$

In the modified version, the objective lens still lands its image at its own focal point (its $d_o$ is still $\infty$), but that’s no longer the focal point of the eyepiece. Rather it’s at an intermediate point, where the inverter then uses it as its object, in turn forming its own (real) image at the focal point of the eyepiece. Thus the modified distance between the objective lens and the eyepiece is: 

$$L_{modified} = f_o + d_{o,inv} + d_{i,inv} + f_e$$

If the inverter lens isn’t to change the overall magnification, it must simply invert the image without altering its size. That is: 

$$m_{inv} = -(d_{i,inv}/d_{o,inv}) = -1$$

Which means: 

$$d_{i,inv} = d_{o,inv}$$

And of course we know this: 

$$1/f_{inv} = 1/d_{o,inv} + 1/d_{o,inv}$$

Which means: 

$$1/f_{inv} = 2/d_{o,inv}$$

Therefore: 

$$d_{o,inv} = d_{i,inv} = 2f_{inv} = 2f_e$$

(because the inverter is identical to the eyepiece)

So: 

$$L_{modified} = f_o + d_{o,inv} + d_{i,inv} + f_e = f_o + 5f_e$$

That is: 

$$f_o + 5f_e = 96.0$$

Solve this and first equation above ($f_o/f_e = 120$) simultaneously for $f_o$: 

$$f_o = 96.0/(1 + 5/120) = 92.2 \text{ cm}$$
Viewing the moon (a distance $X$ away) with a telescope, you see two objects on the lunar surface. Here is some other information about the telescope:

- The telescope’s barrel length (distance between its two lenses) is $L$.
- If you were to place its eyepiece alone next to your eye, it would properly correct your farsightedness.
- If you were to replace its objective lens with another lens identical to its eyepiece, the resulting microscope would have an approximate angular magnification of $M$.

The images of the two lunar objects are separated on your retina by a distance $h_i$. If your eye’s depth (the distance from its lens to retina) is $d_i$, about how far apart are those objects (from each other) on the moon?

**Known values:** $X, L, M, h_i, d_i$

*Note:* This is an ODAVEST items. Use the full 7-step problem-solving protocol—but keep in mind that you’re not being asked to actually solve for the final expression. In fact, you’re not being asked to do any math at all—not even any algebra. Rather, for the Solve step, you are to write a series of succinct instructions on how to solve this problem. Pretend that your instructions will be given to someone who knows math but not physics. And for the Test step, you should consider the situation and predict how the solution would change if the data were different—changing one data value at a time.

**Objective:** Two objects on the moon’s surface are separated by an unknown distance.

The objects are being viewed at a (large) known distance from earth via a telescope.

The telescope’s barrel length (the distance between lenses) is a known value.

The telescope’s eyepiece, if removed and placed next to the eye of this viewer, would properly correct the farsightedness of that eye.

If the telescope’s objective lens were replaced with a lens identical to the eyepiece, the resulting microscope would offer this viewer’s eye a known angular magnification.

The viewer’s eye depth is a known distance.

The images of the two objects on the viewer’s eye’s retina are separated by a known distance.

We want to find the separation distance of the objects on the moon’s surface.

**Data:**

- $X$ The viewing distance from earth to moon.
- $L$ The barrel length (lens-to-lens distance) of the telescope.
- $M$ The angular magnification that would be offered to the viewer’s eye by a microscope constructed from lenses identical to the telescope’s eyepiece.
- $h_i$ The separation distance of the two images that form on the viewer’s retina.
- $d_i$ The depth of the viewer’s eye.

**Assumptions:**

- **Object(s):** We will assume that the two objects are both at the same distance $X$ from the viewer’s telescope.

- **Microscope:** The $M$ value given assumes that the distance between any object viewed and the objective lens’ focal point is negligible.

- **Eye:** We will assume that the eye depth applies equally to each of the two images on the retina.

- **Conditions:** We will assume that the atmosphere does not magnify/reduce/distort the view.
Visual Rep(s):

“Unaided”

“Aided”

Equations:

I. \( \frac{1}{f_e} = \frac{1}{25} + \frac{1}{(-NP)} \)  
   (Correction equation for farsightedness, using the eyepiece.)

II. \( M = -NP(L - f_f)/[(f_f)(f_e)] \)  
   (Approximate angular magnification for a microscope using two identical copies of the eyepiece.)

III. \( f_o = L - f_e \)

IV. \( M_{\text{telescope}} = -f_o/f_e \)

V. \( \theta \approx Y/X \)

VI. \( \theta' \approx h/d_i \)

VII. \( M_{\text{telescope}} = \theta'/\theta \)

Solving:

Solve I for NP. Substitute the resulting expression into II.

Solve II for \( f_e \). Substitute that result into III and IV.

Solve III for \( f_f \). Substitute that result into IV.

Solve IV for \( M_{\text{telescope}} \). Substitute that result into VII.

Substitute V into VII.

Solve VI for \( \theta' \). Substitute that result into VII.

Solve VII for Y.
Testing: Dimensions: Y should have units of length.

Dependencies: Assuming all other data as given, if X were greater, this would imply a greater value of Y.

Assuming all other data as given, the effects of L on Y are not clear without graphing; the values of L, M, NP and f_e are co-dependent in 2nd- and 3rd- order polynomials.

Assuming all other data as given, the effects of M on Y are not clear without graphing; the values of L, M, NP and f_e are co-dependent in 2nd- and 3rd- order polynomials.

Assuming all other data as given, if h_i were greater, this would imply a greater value of Y.

Assuming all other data as given, if d_i were greater, this would imply a lesser value of Y (because the same h_i would imply a lesser θ').
11. a. Suppose you have a 2x and two 20x magnifying glasses. And suppose you want to use those three lenses to build a spyglass (a telescope that gives you a non-inverted view) with an angular magnification of about 30. How long would this spyglass be (measured from objective lens to the eyepiece)?

First, note the focal lengths of the lenses. A magnifier’s “power” rating is simply $25/f$, where $f$ is in cm. Thus, you have lenses with focal lengths of 12.5, 1.25 and 1.25, respectively.

Now, if you were to build a standard two-lens telescope from the long and short lenses (i.e. $f_o = 12.5$ cm and $f_e = 1.25$ cm), where the two focal points coincide ($L = f_o + f_e$), this would offer you an angular magnification of

$$M \approx -f/fe = -12.5/1.25 = -10$$

You want to do 3 times better than that. So the inverter lens (the other available lens; $f_{inv} = 1.25$ cm) must not only invert the image, it must grow it by a factor of three: $m_{inv} = -3$

In other words, both of these equations must be true for the inverter lens:

$$1/f_{inv} = 1/d_{o,inv} + 1/d_{i,inv}$$
$$-(di,inv/do,inv) = -3$$

Solving these simultaneously:

$$1/f_{inv} = 1/d_{o,inv} + 1/(3d_{o,inv})$$
$$1/f_{inv} = 4/(3d_{o,inv})$$
$$d_{o,inv} = (4/3)f_{inv} = (4/3)(1.25) = 1.6667\text{ cm}$$
$$d_{i,inv} = 3d_{o,inv} = 3(1.667) = 5.000\text{ cm}$$

So the length, $L$, of the whole spyglass must be: $L = f_o + d_{o,inv} + d_{i,inv} + f_e$

$$= 12.5 + 1.6667 + 5.000 + 1.25 = 20.4\text{ cm}$$
11. b. As an avid bird-watcher, you’re stalking the rare and elusive* North American Tinky Bird. Unfortunately, your assistant just dropped the binoculars into a pond. And now (wouldn’t you know it?), 200 m away, standing 40 cm tall on the pond’s opposite shore is an unidentified bird. Is it… could it be…? But at that distance, you can’t see enough detail with your naked eye. Drat. So you decide to dismantle your assistant’s eyeglasses (two identical lenses, each with a focal length of 28.29 cm) to try to enhance your view of the mystery bird.

(i) First you use the lenses to construct a standard-configuration 2-lens telescope (you have your assistant hold the lenses the right distance apart while you view the result). Make a quick sketch of this, showing relevant lengths and positions, and then explain why this doesn’t enhance the detail in your view (plus the bird is upside-down—very annoying).

(ii) Then strikes The Bright Idea: Forget the normal assumptions of a telescope. You have your assistant change the distance, \( L \), between the two lenses, and you also re-locate your eye so that you get an enlarged and upright view of the bird. In fact, depending on where you place your eye (it has normal uncorrected vision), you can get an angular magnification value as high as +40.0 with the lenses separated by that distance \( L \). Calculate \( L \).

(iii) At long last, you can positively identify it, get famous and retire: This IS the fabled Tinky. But you need a second person to corroborate your claim. So now you hold the lenses (same separation \( L \) as in part b) and your assistant then views the bird. How far should his eye be from the nearest lens in order to maximize his angular magnification?

(iv) What is the maximum angular magnification your assistant gets with this makeshift system?

*So rare and elusive, in fact, that it’s never been seen. Its existence throughout North America is predicted theoretically by modern physics (but unfortunately, the search is being conducted only in a small region near the French-Swiss border).

(i) The angular magnification of this telescope is \( M \approx \frac{-f_i/f_o}{1} \)

\[
= \frac{-28.29/28.29}{1} = -1.00
\]

That’s no improvement in view:

(ii) With two converging lenses (and \( d_{o,i} > f \)), the only way to create an upright final image is this: The first lens produces an inverted (real) image. The second lens then uses that image as its object, also inverting the resulting image. Your eye must be somewhere to the right of the 2nd image to see it.

Locate the first image: \( 1/f_1 = 1/d_{o,1} + 1/d_{i,1} \) Or: \( 1/28.29 = 1/20000 + 1/d_{i,1} \) So: \( d_{i,1} = 28.3301 \) cm.

Find the first magnification: \( m_1 = -(d_{i,1}/d_{o,1}) = -(28.3301/20000) = -0.0014165 \)

Find the first image height: \( h_{i,1} = m_1(h_o) = -0.05666 \) cm

Your unaided viewing of the bird is given by \( \theta \approx 0.40/200 = 0.00200 \) And your best aided view is 40 times better than this: \( \theta’ \approx 40(0.002) = 0.08 \). But \( \theta’ \approx h_{i,2}/(d_{view}) \), and since you have normal vision, you’d be standing 25 cm to the right of the image to view it. So \( 0.08 = h_{i,2}/25 \), so \( h_{i,2} = 2.00 \) cm.

Therefore: \( m_2 = h_{i,2}/h_{o,2} = h_{i,2}/h_{i,1} = 2.00/(-0.05666) = -35.29818 \)

So now we know: \( -(d_{i,2}/d_{o,2}) = -35.29818 \). And, of course, \( 1/28.29 = 1/d_{o,2} + 1/d_{i,2} \)

Solving simultaneously, we get: \( d_{o,2} = 29.09 \) and \( d_{i,2} = 1029 \)

And therefore: \( L = d_{i,1} + d_{o,2} = 28.3301 + 29.09 = 57.4 \) cm

(iii) Your assistant can’t view the above image as closely as you can; he’s far-sighted (usually corrected with +f eyeglasses worn 2 cm from his eyes). So find his near point—the closest viewing distance he can use—to maximize his angular magnification: \( 1/28.29 = 1/(25–2) + 1/(NP–2) \) Solving: \( NP = d_{view} = 125 \) cm

So he must stand a distance of \( 125 + d_{i,2} \) from that nearer lens: \( 125 + 1029 = 1154 \) cm = \( 11.5 \) m

(iv) Your assistant’s best unaided viewing angle is the same as yours (\( \theta \approx 0.40/200 = 0.002 \)). And the same lens configuration will produce the same final image height (\( h_i = 2.00 \) cm). But his viewing distance is 125 cm (rather than the 25 cm you could use). So for him, \( \theta’ \approx 2/125; \) so his \( M = \theta’/\theta = (2/125)/0.002 = 8.00 \)
11. c. Suppose you have four lenses with focal lengths 2.00 cm, 3.00 cm, 4.00 cm, and 36.0 cm, respectively.

(i) First, select among your four lenses to form the best conventional 2-lens telescope.

\[ M \approx -f_o/f_e \] so to maximize \( M \), you want to maximize \( f_o \) and minimize \( f_e \). So, let \( f_o = 36 \text{ cm} \) and let \( f_e = 2 \text{ cm} \).

When you look through it, of course, your view is enlarged but inverted; you things upside down. So, how would you modify it—change its dimensions and place one of the other lenses somewhere between the objective lens and the eyepiece—to get a non-inverted view with the same angular magnification magnitude as before? (In selecting this third lens, keep the length of the telescope as short as possible.)

Use the 3 cm lens as the inverter lens and place it so that its object is the image of the objective lens, and its image is the object of the eyepiece (placed, of course, at the focal point of the eyepiece). Note that we want the magnification, \( m \), of the inverter lens to be \(-1\) (which flips the image with no change in size).

Thus we need to solve these two equations for the \( d_i \) and \( d_o \) of the inverter lens (whose \( f = 3 \text{ cm} \)):

\[ -\frac{d_i}{d_o} = -1 \] \hspace{1cm} \frac{1}{3} = \frac{1}{d_o} + \frac{1}{d_i} \hspace{1cm} \text{Results:} \hspace{0.5cm} d_o = d_i = 2f = 6 \text{ cm} \\

Draw a simple sketch of the resulting 3-lens instrument.

(ii) With your answer to part (a) as a starting point, keeping the relative order of those three lenses the same, how would you modify their distances from one another in order to approximately double the angular magnification of the overall instrument?

To roughly double the angular magnification of the instrument, we’d simply double the height of the image that the inverter lens delivers to the eyepiece. That is, now we want the magnification, \( m \), of the inverter lens to be \(-2\) (which flips the image while doubling the size).

Thus we need to solve these two equations for the \( d_i \) and \( d_o \) of the inverter lens (whose \( f = 3 \text{ cm} \)):

\[ -\frac{d_i}{d_o} = -2 \] \hspace{1cm} \frac{1}{3} = \frac{1}{d_o} + \frac{1}{d_i} \hspace{1cm} \text{Results:} \hspace{0.5cm} d_o = \frac{3}{2}f = 4.5 \text{ cm} \hspace{0.5cm} d_i = 3f = 9 \text{ cm} \\

Again, draw a simple sketch of your answer.
11. c. (iii) Same sort of question for a microscope: When you look through a conventional 2-lens microscope, your view is inverted—you see the object enlarged but upside down. First, select among your four lenses to form the best conventional 2-lens microscope that has a distance of 50.0 cm between the lenses.

\[ M \approx -NP(L - f_e)/(f_o f_e), \]

so to maximize \( M \), you want to minimize both \( f_o \) and \( f_e \). And for a given \( L \) value, this equation would indicate that you’d want \( f_e \) to be the shortest of all (2 cm). So if \( L \) is to remain at 50 cm (and you could adjust the object distance accordingly), you must initially select \( f_o = 3 \) cm and \( f_e = 2 \) cm.

Now, assuming you’ll be viewing an object located 3.20 cm from the objective lens, how would you then modify this instrument—change its dimensions and place one of the other lenses somewhere between the objective lens and the eyepiece—to get a non-inverted view with the same angular magnification magnitude? (You can change the overall length of the microscope, but keep it as short as possible.)

To invert the image delivered to the eyepiece—with no change in its size—the logic is exactly like part a: But the other short lens available in this case is the 4 cm lens—you’ll have to use that as the inverter lens and place it so that its object is the image of the objective lens, and its image is the object of the eyepiece (placed, of course, at the focal point of the eyepiece). Note that we want the magnification, \( m \), of the inverter lens to be \(-1\) (which flips the image with no change in size).

Thus we need to solve these two equations for the \( d_i \) and \( d_o \) of the inverter lens (whose \( f = 4 \) cm):

\[-(d_i/d_o) = -1 \quad 1/4 = 1/d_o + 1/d_i \quad \text{Results: } d_o = d_i = 2f = 8 \text{ cm}\]

Also, calculate the objective lens’ image distance: \( 1/3 = 1/3.20 + 1/d_i \quad \text{Result: } d_i = 48 \text{ cm}\)

Draw a simple sketch of the resulting 3-lens instrument.

(iv) With your answer to part (iii) as a starting point, keeping the relative order of those three lenses the same, how would you modify their distances from one another in order to approximately triple the angular magnification of the overall instrument?

To roughly triple the angular magnification of the instrument, we’d triple the height of the image that the inverter lens delivers to the eyepiece. That is, now we want the magnification, \( m \), of the inverter lens to be \(-3\) (which flips the image while tripling the size). Thus we need to solve these two equations for the \( d_i \) and \( d_o \) of the inverter lens (whose \( f = 4 \) cm):

\[-(d_i/d_o) = -3 \quad 1/4 = 1/d_o + 1/d_i \quad \text{Results: } d_o = (4/3)f = 5.33 \text{ cm} \quad d_i = 4f = 16 \text{ cm}\]

Again, draw a simple sketch as part of your answer.
12. You’re on your way out of town for spring break when a sudden freak thunderstorm whisks you to a faraway desert island. Alas. Fortunately, washing up on the beach in the storm’s aftermath is a watertight bag containing lots of food and your favorite beverages, a tub of industrial-strength sunscreen, a hammock and canopy, plus three converging lenses, a small flat mirror, a meter stick, a calculator and your PH 212 notes sheet. No smart phone. No dumb phone. No laptop. No iPad. No iPod. No iNothing. All there is to do is physics until you’re rescued. Alas.

a. First you determine the focal lengths of the lenses (3.00 cm, 5.00 cm and 40.0 cm). Explain how you do this (no equations or math necessary).

The sun’s rays are parallel, and a converging lens concentrates (converges) parallel rays at its focal point. So take each lens in turn and experiment with it until you find how far from the lens is the point that is (very) hot and can start a fire or burn a hole in something. Use the meter stick to measure that distance.

b. Your notes sheet is seriously over-crammed with tiny writing—a magnifying glass would sure help. If the distance from your eye lens to your retina is 1.92 cm, and you have normal vision (i.e. not needing correction), then by using just one of the three lenses, what’s the approximate height of the largest letter E you could form on your retina, if you wrote that E in your notes just 1.60 mm tall?

The best angular magnification you can get with a converging lens as a magnifying glass is:  

\[ M = \frac{NP}{f} + 1 \]

M is highest when the focal length is the shortest, so use the lens with \( f = 3 \) cm. Thus:

\[ M = \frac{25}{3} + 1 = 9.333 \]

The best unaided view, \( \theta \), you’ll get of the E is from your near point (\( NP = 25 \) cm):

\[ \theta \approx \frac{1.60 \times 10^{-3}}{0.25} \approx 6.40 \times 10^{-3} \text{ rad} \]

And this is the same angle formed by the image height, \( h_{i,\text{eye}} \), on your retina, vs. the eye depth, \( d_{i,\text{eye}} \):

But the magnified view, \( \theta' \), is \( M \) times this good:

\[ \theta' \approx M\theta \approx 9.333(6.40 \times 10^{-3} \text{ rad}) \approx 5.973 \times 10^{-2} \text{ rad} \]

That’s the angle formed by the image height, \( h_{i,\text{eye}} \), on your retina, vs. the eye depth, \( d_{i,\text{eye}} \):  

\[ \theta' \approx h_{i,\text{eye}}/0.0192 \]

Solve for \( h_{i,\text{eye}} \):

\[ h_{i,\text{eye}} \approx 0.0192(0.0192(5.9733 \times 10^{-2})) \approx 1.15 \times 10^{-3} \text{ m} \quad \text{(or 1.15 mm)} \]
12. c. Not satisfied with just a simple magnifying glass, you choose two lenses, holding one next to your eye and the other lens anywhere within your 45-cm reach. Some pairs of lenses produce better magnification results than other pairs. With the best such pair chosen, what’s the size of the E on your retina now?

You’re trying to form the best microscope you can with any two of the lenses you have. The angular magnification for a microscope is:

$$M = -NP(L - f_e)/(f_o f_e)$$

This value is highest when $L$ is longest, $f_e$ is shortest, and $f_o$ is the next-shortest. So use the 3-cm lens as the eyepiece; the 5-cm lens as the objective; and hold them at maximum separation—full arm’s length (45 cm):

Then:  
$$M = -NP(L - f_e)/(f_o f_e) = -25(45 - 3)/(3 \cdot 5) = -70.000$$

Again, the best unaided view, ϑ, you’ll get of the E is $\theta \approx 6.40 \times 10^{-3}$ rad, just as with the magnifying glass — see diagram of ϑ from part (b).

But the magnified view, $\theta'$, is $M$ times this good:  
$$\theta' \approx M\theta \approx -70(6.4000 \times 10^{-3} \text{ rad}) \approx -0.448 \text{ rad}$$

The same angle is formed by the image height, $h_{i,\text{eye}}$, on the retina, vs. the eye depth, $d_{i,\text{eye}}$:  
$$\theta' \approx h_{i,\text{eye}}/0.0192$$

Solve for $h_{i,\text{eye}}$:  
$$h_{i,\text{eye}} \approx 0.0192(\theta') = 0.0192(-0.448) \approx -8.60 \times 10^{-3} \text{ m} \text{ (or } -8.60 \text{ mm)}$$

(The only problem: it’s upside down.... Guess you’ll have to stick with the magnifying glass to actually read your notes.)
12. d. Out on the horizon (which you know is \(\approx 20\) km away), something appears that might be a boat (!), but you can’t tell for sure—it’s just a tiny dot on your retina. Ah, but now that you can read your notes sheet, you can figure out the best pair of lenses to see more detail: Using that combination, you now see a 0.100 mm image on your retina of this vessel. About how tall is this boat?

You’re trying to form the best telescope you can with any two of the lenses you have.

The angular magnification for a telescope is:

\[ M = -\frac{f_o}{f_e} \]

This value is highest when \(f_e\) is shortest, and \(f_o\) is the longest—and note that the separation of these two lenses must equal the sum of their two focal lengths:

\[ L = f_o + f_e. \]

So use the 3-cm lens as the eyepiece; the 40-cm lens as the objective; and hold them at a separation equal to the sum of those two focal lengths (43 cm):

Thus: \( M \approx -\frac{f_o}{f_e} = -\frac{40}{3} = -13.333 \)

Compare the above (aided) situation to the unaided (“naked-eye”) situation below:

By definition, \( M = \theta'/\theta \), and for this situation:

\[ M \approx \left[\frac{h_{i,\text{eye}}}{d_{i,\text{eye}}}\right]/\left[\frac{h_i}{d_o}\right] \]

Solving for \( h_o \):

\[ h_o = \frac{(d_o h_{i,\text{eye}})}{(M d_{i,\text{eye}})} \]

The numbers:

\[ h_o = \frac{[20,000(0.100 \times 10^{-3})]/[(-13.333)(0.0192)]} = (-)7.81 \text{ m} \]

(Yes, it’s upside down, but at least you can see that it’s a boat—at least a decent-sized yacht, apparently: it rides about 25 feet out of the water.)
12. e. As you observe the boat, it’s not moving toward you. Alas. Hmmm.... The sun is shining down on you, making an angle of 70° with the level ground—coming from slightly behind you (over your shoulder) as you face out to sea toward the boat. At what angle with respect to the ground should you hold the mirror to signal the boat?

You want the sunlight (angled as shown) to be incident on the mirror so that the reflected ray is parallel to the ground.

The key items are the normals to both the ground and the mirror--here’s the complete picture:
The question is: What is the measure of the angle \( \alpha \)?

As with all reflections, \( \theta_i = \theta_r \).
And, using the plane of the level ground as 180°:
\[
\theta_i + \theta_r + 70 = 180
\]
Solving these, we get:
\[
\theta_i = \theta_r = 55°
\]
Then, note that \( \beta + \theta_i = \beta + \theta_r = 90° \), so \( \beta = 35° \).
But \( \beta + \alpha = 70° \), so \( \alpha = 35° \), also.

You should hold the plane of the mirror at an angle of 35° as measured from the ground.