Prep 3

*Recommended finish date: Friday, January 25*

The formats (type, length, scope) of these Prep problems have been purposely created to closely parallel those of a typical exam (indeed, these problems were taken from past exams). *To get an idea of how best to approach various problem types (there are three basic types), refer to these Sample Problems.*
1. a. Evaluate the following statements (T/F/N). As always, you must explain your reasoning with a valid mix of words, diagrams and/or calculations.

(i) If a steady net torque $\tau$ (a known value) is applied to an object (initially at rest) about some fixed axis of rotation, for a known time interval, $\Delta t$, and its resulting angular speed is measured as $\omega$ at the end of that time interval, then this is enough information to calculate the object’s moment of inertia about that axis of rotation.

(True) We can calculate $a$ via $w_f = w_i + a\Delta t$ (where $w_i = 0$); then we get $I$ via $\mathcal{S}t = Ia$.

(ii) For a moving object, it is possible that $\Sigma F = 0$ and $\Sigma \tau \neq 0$.

(iii) For a moving object, it is possible that $\Sigma F = 0$ and $\Sigma \tau = 0$.

(iv) It is possible that an object moving (translating) in a circle might have no forces acting on it.

(v) A rotating object can be in total mechanical equilibrium.

(vi) If the gravitational force on an object produces zero torque about a certain axis, the object’s center of gravity is located on the same vertical line as that axis.

(vii) Torque is a vector quantity.

(viii) A radial force is a constant vector value.

b. Express torque in fundamental (base) SI units.

c. You are facing a wheel initially at rest. A force is applied to the right edge of the wheel, accelerating the wheel in a clockwise direction. The direction of the torque vector here is...

d. Exercise 20, page 349.

The disk shown here is 20 cm in diameter and is free to rotate on a frictionless axle at its center (perpendicular to the page). Points A and B are each 5.0 cm from the axle. Find the net torque on the disk.
2. a. The second hand on a certain standard clock has a length of 12.0 cm, measured from the central hub to its tip. Normally, that second hand sweeps smoothly and steadily around one full revolution every minute (60 seconds). Suppose that it is indeed operating normally—until a certain moment, when it begins to speed up....

(i) What angular acceleration would it need to have—at that first moment of speeding up—so that the tip of the second hand then has equal magnitudes of tangential and radial acceleration?

(ii) Assuming the behavior described in part (i), find the total acceleration magnitude of the tip at that first moment of speeding up.

(iii) Assuming the behavior described in part (i), if the tip of the second hand is located at the “4 o’clock” position at that first moment of speeding up, find the direction of its total acceleration, $a$, at that moment.
2. b. At the moment depicted here, this roller coaster car of mass \( m \) is at the angular position \( \theta \), sliding clockwise at speed \( v \), in a vertical "loop-the-loop" maneuver on this circular track of radius \( r \), which has a kinetic friction coefficient \( \mu_K \) with the car.

Find an expression for the net force (both magnitude and direction) acting on the car.

You may consider these values as known: \( m, \theta, v, r, \mu_K, g \).
2. c. A record with a 10 cm radius and 200 g mass is dropped vertically (and without rotation) onto a turntable that is rotating but not being driven by a motor; rather, it is gradually slowing, due to a constant net torque of 0.01 N·m exerted by the friction in its hub. When the record lands on the turntable, the two objects stick together and rotate around the turntable’s hub. The radius of the turntable is 10 cm; its mass is 2 kg; its angular velocity just before the record lands on it is 5 rad/s.

(i) What is earliest common angular speed achieved by the record and the turntable?

(ii) How long will it take for the record and turntable to stop rotating altogether?

d. A uniform rod (2.50 m long, 41.3 kg) is rotating in a horizontal circle about one end, as shown in this overhead view. When the rod is in the position shown, it is rotating at –6.7 rad/s, and a torque of 89.0 N·m is being applied to it. What is the direction of the net force being exerted on the outer tip of the rod at that instant?
A horizontal turntable is a disk free to rotate frictionlessly around its central axis. Initially it is at rest, and a coin is resting on it at position A (a distance $r_A$ from the axis), as shown here. Then a constant torque $\tau$ is applied to the turntable (around its central axis) for a certain time interval, $\Delta t$. At the end of $\Delta t$, the torque ceases.

At that very same moment (the end of $\Delta t$), the coin slips and begins to slide across the surface of the turntable. It stops slipping at position B (a radial distance $d$ from A), as shown, because it encounters an obstruction in the turntable surface.

The mass of the coin is $m$. The moment of inertia of the turntable (without the coin), around its central axis, is $I$. After the coin has arrived at position B (after it has stopped slipping), the friction force on the coin is half the maximum possible. Find the steady force magnitude exerted by the obstruction on the coin after it has arrived at position B (after it has stopped slipping).

You may consider these values as known: $r_A$, $\tau$, $\Delta t$, $d$, $m$, $I$, $g$.

This is an ODAVEST item—use the full seven-step problem-solving protocol—but keep in mind that you’re not being asked to actually solve for the final expression. In fact, you’re not being asked to do any math at all—not even any algebra. Rather, for the Solve step, you are to write a series of succinct instructions on how to solve this problem. Pretend that your instructions will be given to someone who knows math but not physics. And for the Test step, you should consider the situation and predict how the solution would change if the data were different—changing one data value at a time.
I. \[ I_A = I + mr^2 \]

II. \[ t = IAa \]

III. \[ w_A = wi + aDt \]

IV. \[ F_{max} = m(rAw^2) \]

V. \[ IB = I + m(rA + d)^2 \]

VI. \[ IAw_A = IBw_B \]

VII. \[ F_{obstr.} + F_{max}/2 = m[(rA + d)w_B^2] \]

Solving:
1. Solve I for \( I_A \). Substitute that result into II.
2. Solve II for \( a \). Substitute that result into III.
3. Solve III for \( w_A \) (noting that \( wi = 0 \)). Substitute that result into VI.
4. Solve IV for \( F_{max} \). Substitute that result into VII.
5. Solve V for \( IB \). Substitute that result into VI.
6. Solve VI for \( w_B \). Substitute that result into VII.
7. Solve VII for \( F_{obstr.} \).

Testing:
- Dimensions:
  - \( F_{obstr.} \) should have units of force (dimensions of mass·length/time$^2$).
- Dependencies:
  - If the coin starts farther from the axle (a greater \( r_A \)), then for the given torque, \( w_A \) will be less, due to the larger moment of inertia. That implies a lesser value for \( F_{max} \), but since \( w_B \) would also be less, it's unclear without an explicit solution how all this would affect \( F_{obstr.} \).
  - If the torque magnitude is greater, then \( w_A \) will be greater, which would imply a stronger \( F_{max} \). But since \( w_B \) would also be greater, it's unclear without an explicit solution how all this would affect \( F_{obstr.} \).
  - If the torque interval, \( Dt \), is greater, then \( w_A \) will be greater, which would imply a stronger \( F_{max} \). But since \( w_B \) would also be greater, it's unclear without an explicit solution how all this would affect \( F_{obstr.} \).
  - If the coin's sliding distance, \( d \), is greater, this will reduce \( w_B \) and therefore the net centripetal force acting on the coin at point B. That would imply a lesser role necessary for \( F_{obstr.} \).
  - If the coin's mass, \( m \), were greater, then for the given torque, \( w_A \) will be less, due to the larger moment of inertia. That implies a lesser value for \( F_{max} \), but since \( w_B \) would also be less, it's unclear without an explicit solution how all this would affect \( F_{obstr.} \).
  - If the carousel has a greater moment of inertia, \( I \), then for the given torque, \( w_A \) will be less. That implies a lesser value for \( F_{max} \), but since \( w_B \) would also be less, it's unclear without an explicit solution how all this would affect \( F_{obstr.} \).
  - If the local g value were greater, this would increase the value of \( F_{max} \), so the obstruction force, \( F_{obstr.} \), would play a lesser role at point B.
2. f. Refer to the overhead view here. A large, uniform disk (mass = \( m_1 \), outer radius = \( r_1 \)) is rotating freely on a frictionless central axis. A small coin (mass = \( m_2 \)) is “riding” on the disk at a distance \( d \) from the axis. The coin’s speed (relative to the ground) is \( v \), and it does not slip across the disk’s surface. The static friction coefficient between the coin and disk surfaces is \( \mu_s \). Ignore any effects of wind/air drag.

(i) What is the maximum steady torque magnitude you could apply to the disk (without touching the coin) in order to bring the disk (and coin) to rest without the coin ever slipping across the disk’s surface?

(ii) How much time would this slowing-to-a-stop process require?

Here is a summary of all known values: \( m_1, r_1, m_2, d, \mu_s, v, g \)

For this item, the full seven-step ODAVEST problem-solving procedure is required. Note that each part will be awarded with points, so even if you don’t get all the way through the problem, there are many ways to earn partial credit for parts that are valid.

Keep in mind that you’re not being asked to actually solve for the final expression. In fact, you’re not being asked to do any math at all—not even any algebra. Rather, for the Solve step, you are to write a series of succinct instructions on how to solve this problem. Pretend that your instructions will be given to someone who knows math but not physics. And for the Test step, you should consider the situation and predict how the solution would change if the data were different—changing one data value at a time.
Assumptions:

Disk
We assume that the disk is perfectly balanced on its axle, so that there is no "wobble" or vibration as it rotates.

Coin
We assume that the coin is small enough to model as a point mass on the surface of the large disk.

Surfaces
We assume that the given coefficient of static friction applies equally in all horizontal directions for the disk—that its surface has no "grain" or bias. We assume that the given coefficient of static friction applies equally in all horizontal directions for the coin—that its surface has no "grain" or bias.

Torque
We assume that the sudden application of the steady torque causes no wobble.

Speed
We assume that the given speed \( v \) is less than the maximum steady speed at which the coin could ride without slipping.

Visual Representation:

Equations:

I. \( \tau_{\text{max}} = I_{\text{total}} a_{\text{max}} \)

II. \( I_{\text{total}} = \frac{1}{2} m_1 r_1^2 + m_2 d_2^2 \)

III. \( v = dw \)

IV. \( |a_R|_{\text{max}} = dw^2 \)

V. \( |a_T|_{\text{max}} = da_{\text{max}} \)

VI. \( a_{\text{max}} = \sqrt{|a_R|_{\text{max}}^2 + |a_T|_{\text{max}}^2} \)

VII. \( F_{\text{S.12 max}} = m_2 a_{\text{max}} \)

VIII. \( F_{\text{S.12 max}} = m_2 F_{\text{N.12}} \)

IX. \( F_{\text{N.12}} = m_2 g \)

X. \( \omega = (0) + a_{\text{max}} (D_{\text{stop.min}}) \)
Solving: Solve II for $I_{\text{total}}$. Substitute that result into I.

Solve III for $w$. Substitute that result into IV and X.

Solve IV for $|a_R|_{\text{max}}$. Substitute that result into VI.

Solve IX for $F_N$. Substitute that result into VIII.

Solve VIII for $F_{S\text{max}}$. Substitute that result into VII.

Solve VII for $a_{\text{max}}$. Substitute that result into VI.

Solve VI for $|a_T|_{\text{max}}$. Substitute that result into V.

Solve V for $a_{\text{max}}$. Substitute that result into I and X.

Solve I for $t_{\text{max}}$.

Solve X for $D_{t\text{stop.min}}$.

Testing:

Dimensions: $t_{\text{max}}$ should have dimensions of force·length.

$D_{t\text{stop.min}}$ should have dimensions of time.

Dependencies:

If $m_1$ were greater, then with all other variables the same, this would not change $a_{\text{max}}$, thus not affect $a_{\text{max}}$. But it would produce a larger $I_{\text{total}}$. So: $t_{\text{max}}$ would be greater, but $D_{t\text{stop.min}}$ would not change.

If $r_1$ were greater, then with all other variables the same, this would not change $a_{\text{max}}$, thus not affect $a_{\text{max}}$. But it would produce a larger $I_{\text{total}}$. So: $t_{\text{max}}$ would be greater, but $D_{t\text{stop.min}}$ would not change.

If $m_2$ were greater, then with all other variables the same, this would not change $a_{\text{max}}$, thus not affect $a_{\text{max}}$. But it would produce a larger $I_{\text{total}}$. So: $t_{\text{max}}$ would be greater, but $D_{t\text{stop.min}}$ would not change.

If $d$ were greater, then with all other variables the same, this would produce smaller values for $w$ and for $|a_T|_{\text{max}}$, which would allow a greater value for $|a_T|_{\text{max}}$ (since $F_{S\text{max}}$ is unchanged), thus a greater $a_{\text{max}}$. So: $t_{\text{max}}$ would be greater, and $D_{t\text{stop.min}}$ would be smaller.

If $m_S$ were greater, then with all other variables the same, this would mean $F_{S\text{max}}$ would be greater, allowing a greater $a_{\text{max}}$. This would allow a greater $|a_T|_{\text{max}}$, thus a greater $a_{\text{max}}$. So: $t_{\text{max}}$ would be greater, and $D_{t\text{stop.min}}$ would be smaller.

If $v$ were greater, then with all other variables the same, this would produce greater values for $w$ and for $|a_T|_{\text{max}}$, which would require a smaller “allowance” for $|a_T|_{\text{max}}$ (since $F_{S\text{max}}$ is unchanged), thus a lesser $a_{\text{max}}$. So: $t_{\text{max}}$ would be smaller, and $D_{t\text{stop.min}}$ would be greater.

If $g$ were greater, then with all other variables the same, this would mean $F_{S\text{max}}$ would be greater, allowing a greater $a_{\text{max}}$. This would allow a greater $|a_T|_{\text{max}}$, thus a greater $a_{\text{max}}$. So: $t_{\text{max}}$ would be greater, and $D_{t\text{stop.min}}$ would be smaller.
3.  

   a. Early peoples exploited the effects of Newton’s laws when they discovered a fulcrum could be used for mechanical advantage. To lift a large 200-kg-boulder, a 3-m-long board is setup with a fulcrum. If a person weighing 750 N stands on the opposite end, where along the board (d) must the fulcrum be placed to lift the boulder. Assume the mass of the board is negligible and the maximum force the person can apply is simply his (resting) weight.

```
\[St = Ia = 0\]
\[(200)(9.80)(d)\sin90° - (750)(3-d)\sin90° = 0\]
\[1960d - (2250 - 750d) = 0\]
\[2710d - 2250 = 0\]
\[d = \frac{2250}{2710} = 0.830 \text{ m} \]
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   (That's 83.0 cm from the end holding the boulder.)

   b. A long, narrow, heavy but uniform board rests on the ground. To just lift one end off the ground with a vertically-directed force while the other end stays on the ground, you effectively have to lift half the board’s weight. You continue to lift the one end of the board until it makes a 40° angle with the ground. If your force at this point is perpendicular to the board itself, how much force (expressed as a fraction of the board’s weight) must you now supply to hold the board at that angle?

```
\[St = Ia = 0\]
\[(F)(L)\sin90° - (mg)(L/2)\sin90° = 0\]
\[(F)(L)\sin90° - (mg)(L/2)\sin50° = 0\]
\[FL - mg\cdot\sin50°\cdot\frac{L}{2} = 0\]
\[F = \frac{mg}{2}\]
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   (or: \[F = \frac{mg\cos40°}{2}\])

   c. Exercise 30, page 349.
   The 1.00-kg board is 2.00 m long. The 4.00-kg block is 1.00 m long. Find the distance \(d\) locating the fulcrum to balance the situation shown.

![Diagram of a fulcrum with a 1.00 kg and a 4.00 kg block on a 2.00 m board]
3.  d.  This uniform, 10-kg board is 4.00 m long and is at rest. Find the distance from its fulcrum to its right end.

The board is uniform, so its center of gravity is the geometric center of the board (midway between ends). The fulcrum is located at an unknown distance \( l \) from the board’s right end. And the board is in (static) mechanical equilibrium: \( \sum F_y = 0 \). Sum the torques about the fulcrum axis:

\[
\sum F_y = I_a F G (2–l) – 49 \sin 30° (l) = I_a mg (2–l) – 49 \sin 30° (l) = 0
\]

Solve for \( l \):

\[
2 mg = l mg + 49 \sin 30° (l)
\]

\[
2 mg = l (mg + 49 \sin 30°)
\]

\[
\frac{2 mg}{mg + 49 \sin 30°} = l = 1.60 \text{ m}
\]

3.  e.  A uniform board has a mass of 35 kg and sits horizontally at rest, supported as shown, by two scales that measure vertical force only (but the scales can exert both vertical and horizontal forces). The force shown is 107 N and is applied directly over scale B. The reading on scale B is 294 N. How long is the entire board?

\[
\sum F_y = F_B.y + F_A.y + F_G = m a_y F_A.y + 294 – 107 \sin q – mg = 0
\]

\[
F_B.y = 364.66 \text{ N}
\]

Find center of board (i.e. cg): sum torques \( \sum \tau = 0 \) about the point above scale B.

\[
F_G (\sin 90°) l_G – F_A.y (\sin 90°) l_A = I_B a_B
\]

\[
mg (l_G) – 141.66 (4.26) = 0
\]

\[
l_G = 1.759 \text{ m}
\]

Board length:

\[
L = 2(4.26 + 1.35 – l_G) = 7.70 \text{ m}
\]

f.  A uniform beam, sitting at rest, has a mass of 53 kg and length of 8.6 m. It is supported as shown, by two scales that measure vertical force only (but the scales can exert both vertical and horizontal forces). The force shown is 170 N and is applied at the point shown. The reading on scale A is 294 N. How far is scale B from the right end?

\[
\sum F_y = F_B.y + F_A.y + F_G = m a_y F_B.y + 294 – 170 \sin 55° – mg = 0
\]

\[
F_B.y = 364.66 \text{ N}
\]

Sum torques about A:

\[
\sum \tau = I_A a_A – F_G [4.62 – 1.53] – 170 (\sin 55°) (4.62) + F_B.y \cdot l_B = 0
\]

\[
– mg (2.77) – 170 (\sin 55°) (4.62) + 364.66 \cdot l_B = 0
\]

\[
l_B = 5.710 \text{ m}
\]

\[
d_B = 8.6 – (l_B + 1.53) = 1.36 \text{ m}
\]
3. g. A uniform board of mass $m$ is propped between two parallel, vertical walls that are separated by a distance $d$, as shown here in this “edge-on” view. Wall 1 is frictionless, but Wall 2 has static friction (coefficient $\mu_s$) with the board. The board length, $L$, is the maximum length possible so that the board does not slip. Let $F_{N1}$ and $F_{N2}$ denote the normal forces exerted by each wall, respectively, on the board.

Evaluate (T/F/N) each statement. Justify your answers fully with any valid mix of words, drawings and calculations.

(i) $F_{N1} = F_{N2}$

(ii) $\mu_s F_{N1} = mg$

(iii) $mgd/2 = (F_{N1})\sqrt{(L^2 - d^2)}$
4. a. Blocks with masses \(m_1 = 30.0 \text{ kg}\) and \(m_2 = 40.0 \text{ kg}\) are connected by a string (with negligible mass) that passes over a pulley. The pulley is a solid disk that has a radius of 6.00 cm and a mass of 10.0 kg.

The string is taut at all times, and it does not slip as it passes over the pulley. The inclined ramp is frictionless.

Find the magnitude of the acceleration of \(m_1\).

*Be sure to do a FBD and use Newton’s 2nd Law for each of the three masses!*
4. b. A fish is hooked on a fishing line and is pulling on it with a constant force, trying to escape. The line is wound around a reel, which is a solid disk (r = 0.213 m; m = 654 kg). At first, the reel is held motionless by the fisherman, who exerts a perpendicular force, \( F \), of 98.7 N on the 1.02 m (massless) handle attached to the hub of the reel, as shown. Suddenly, the entire handle breaks off of the reel, allowing it to spin freely in response to the fish’s pull. Assuming the fish maintains the same steady tension in the line as it swims away, how far will it have swum (directly away from the reel) when the reel’s angular speed is 54.3 rad/sec?

\[ F \cdot d_{\text{handle}} \sin(90°) – F_T R \sin(90°) = 0 \] (where \( R \) is the radius of the reel). Solve this for \( F_T \):

\[ F_T = \frac{F \cdot d_{\text{handle}}}{R} \]

Part 2:

\[ I a – F_T R = \frac{MR^2}{2} a \]

Solve this for \( a \): ...

\[ a = -2 \frac{F_T}{MR} \] (Keep just the magnitude—let the outbound direction be positive.)

Substitute from Part 1:

\[ a = 2 \frac{F \cdot d_{\text{handle}}}{MR^2} \]

Now do kinematics. You know 3 values: \( w_0 \) (= 0), \( w \) (= 54.3) and \( a \) (just solved for).

\[ w^2 = w_0^2 + 2 aDq \]

Rearranged to solve for \( Dq \):

\[ Dq = \frac{w^2 – w_0^2}{2 a} \]

And the amount of line paid out is just

\[ Ds = R Dq = R \frac{w^2 – w_0^2}{2 a} \]

Substitute for \( a \):

\[ Ds = \frac{MR^3 (w^2 – w_0^2)}{4 (F \cdot d_{\text{handle}})} \]

\[ = \frac{(654)(0.213)^3 (54.3)^2 – (0)^2}{4(98.7)(1.02)} \]

\[ = 46.3 \text{ m} \]
4. c. A small child amuses himself by repeatedly slamming his bedroom door, which has a total moment of inertia (about its hinged edge) of \( I \). Each time, he starts with the door at rest—open at the same angle \( \theta \), as shown—then exerts the same steady push \( F \) at the same point, a distance \( d \) from the door’s hinged edge (and he follows the swinging door, so his push is always at right angles to the door) all the way until it slams shut.

Of course, this delights him but annoys everyone else—and it soon damages the door. So now the knobs/lock set (of mass \( m \), located at a distance \( L \) from from the hinged edge), must be removed from the door.

(i) How far from the hinged edge must the child now push on the door (again, always perpendicularly) with the same force \( F \) as before (again, starting with the door at rest at angle \( \theta \)) to get the same impact speed)?

(ii) Comparing the two scenarios (whole door vs. door without the knobs/lock):

   What is the difference in the speed of the child’s hand (which is still pushing on the door) at the moment of impact?

Ignore air drag and hinge friction.

Here are the data: \( F = 20.0 \text{ N}; \ I = 5.00 \text{ kg}\cdot\text{m}^2; \ \theta = 60^\circ; \ d = 65.0 \text{ cm}; \ m = 2.00 \text{ kg}; \ L = 80.0 \text{ cm}.\)
4. d. A solid, rigid, uniform disk of mass $M$ and outer radius $R$ is positioned horizontally over level ground and attached to a thin, frictionless, fixed vertical axle shaft at point X, as shown in this overhead view. (The axle shaft is planted vertically in the ground.)

Point A on the disk is located as shown, at a distance $d$ from the center of the disk. An additional point mass $m$ is located there.

Point B is located on the rim of the disk diametrically opposite to the rotation axis X. No extra mass is located there.

With the disk initially at rest, a [steady] torque $\tau$ is applied to it (about point X) for a time interval $\Delta t$. Then $\tau$ is removed and the disk is allowed to spin freely about its axle at point X.

You may ignore all friction and air resistance. (And the details of how the torque $\tau$ is applied are not needed. That is, you do not need to assume any particular point on the disk where an external force is being applied to produce the torque $\tau$. The value of the torque is sufficient.)

Here are the values you may treat as known: $M, R, d, m, \tau, \Delta t$

Your answers to each of the following should be an expression containing only known values.
(Or, if you don’t want to do the algebra, you may instead list each equation you need—in order—and describe what to solve for in each (much like the Equations and Solve steps in the ODAVEST protocol).

(i) How many revolutions does the disk rotate in the time interval $\Delta t$?

\begin{itemize}
  \item[(i)] How many revolutions does the disk rotate in the time interval $\Delta t$?
\end{itemize}
(ii) Find the magnitude of the net force acting on the point mass $m$ at A at the very end of the time interval $\Delta t$ (while $\tau$ is still being exerted).

(iii) After $\tau$ is removed and the disk is spinning freely, suppose that $m$ breaks loose from point A and rolls out to become stuck at point B instead. What is the speed of mass $m$ now (relative to the ground)?
A jet engine (mass \( m \)) that produces a constant thrust force is mounted for testing on the end of a pivoting arm (length = \( L \); moment of inertia = \( I_{\text{arm}} \)), so that the engine pushes itself around in a horizontal circle with its thrust force, which is perpendicular to the pivot arm, as shown here.

At first, there is a perpendicular braking force applied as shown, at the 3/4 position on the pivot arm, to allow the engine to warm up while still at rest. But when the engine has reached its full operating thrust, the brake is released, and the pivot arm and engine are allowed to rotate freely (with no friction or air resistance).

If the pivot and engine turn \( n \) revolutions in the first \( t \) seconds of motion, what was the force applied by the brake?

Assume these are known values: \( m, L, I_{\text{arm}}, n, t \)
I. $Dq = 2p$

II. $Dq = w_0(t) + \frac{1}{2}a(t)^2$

III. $I_T = I_{arm} + mL^2$

IV. $t_{engine} = I_T a$

V. $t_{engine} - (F_{brake} \cdot \sin90°)(0.75L) = 0$

Solving: Solve I for $Dq$. Substitute that result into II. Solve II for $a$ (noting that $w_0 = 0$). Substitute that result into IV. Solve III for $I_T$. Substitute that result into IV. Solve IV for $t_{engine}$. Substitute that result into V. Solve V for $F_{brake}$.

Testing:

Dimensions: $F_{brake}$, should have dimensions of force (mass·length/time$^2$).

Dependencies: If the mass, $m$, of the engine were greater, that would imply a greater thrust capability overcoming more total moment of inertia in order to achieve the same number of revolutions in the same time. Thus, the braking force, $F_{brake}$, would need to be greater.

If the length, $L$, of the arm were greater, that would imply a greater thrust capability overcoming more total moment of inertia in order to achieve the same number of revolutions in the same time. Thus, the braking force, $F_{brake}$, would need to be greater.

If the moment of inertia, $I_{arm}$, were greater, that would imply a greater thrust capability overcoming more total moment of inertia in order to achieve the same number of revolutions in the same time. Thus, the braking force, $F_{brake}$, would need to be greater.

If the time, $t$, spent rotating under full thrust were greater, that would imply a smaller thrust capability used to accomplish the same number of revolutions. Thus, the necessary braking force, $F_{brake}$, would also be smaller.

If the number, $n$, of revolutions accomplished while rotating under full thrust were greater, that would imply a greater thrust capability in order to achieve this in the same time. Thus, the braking force, $F_{brake}$, would need to be greater.
5. a. Evaluate (T/F/N) this statement (and justify your answers fully with any valid mix of words, drawings and calculations): An object rotating at speed ω about its center of mass has less rotational kinetic energy than if it’s rotating at that same ω about any other parallel axis.

b. A rotating skater draws her arms in, changing her moment of inertia by –8.50% (i.e. reducing it by 8.50%).
   (i) By what percentage does she change her rotational kinetic energy, \( K_r \)?
   (ii) Where does this extra energy come from?

c. In a test laboratory, an electric engine is used to accelerate a wheel from rest. The center of mass of the wheel remains stationary at all times. The wheel’s angular acceleration is in the counter-clockwise direction. It rotates through an angular distance \( Δθ \), achieves a final angular speed \( ω \) and has a constant moment of inertia \( I \). If you assume constant angular acceleration and ignore friction and air resistance, what is the average power output of the engine, expressed in terms of \( Δθ \), \( ω \) and/or \( I \)?

d. One end of a thin, uniform rod, 1.40 m in length, is attached to a pivot. The rod is free to rotate about the pivot without friction or air resistance. Initially, it is hanging straight down (i.e. in the “6 o’clock” position). Then the lower tip of the rod is given an initial horizontal speed \( v_{ti} \), so that the rod rotates upward, about its pivot. Find the value of \( v_{ti} \) so that the rod just reaches the “12 o’clock” position (i.e. straight up), coming to a momentary halt there.

e. A dead tree is initially stationary and there is a 30.0-degree angle between the tree and the vertical. The roots finally give way and the tree topples to the ground (rotating on an axis that is parallel to the level ground and intersecting the base of the tree perpendicularly through the base of the tree). The length of the tree is 30.0 m. How fast is the top of the tree moving (in m/s) just before it hits the ground? Model the tree as straight, rigid rod, and ignore friction and air resistance.
Two identical solid disks are rotating independently about the same thin, frictionless rod. That rod goes through the center of disk A, but in disk B, the rod goes through a point midway between that disk’s center and its outer edge (see overhead view here). Initially, the disks are rotating about the rod with the same angular speed but in opposite directions. Then they collide (without outside interference or any change in altitude) and they stick together as a result. Evaluate (T/F/N) each statement. Justify your answers fully with any valid mix of words, drawings and calculations.

(i) Disk B reverses its direction of rotation as a result of the collision.

(ii) As a result of the collision, $\Delta L_A = -\Delta L_B$.

(iii) About 96.0% of the disks’ total initial mechanical energy is lost as a result of the collision.
5. g. A merry-go-round (carousel) with a moment of inertia of 594 kg·m², is initially rotating horizontally at 2 rad/s about a frictionless axis through its center. Then a metal plate located at that center loosens and gradually slides outward along the floor of the merry-go-round. When the plate is 1.4 m from the center, the merry-go-round has lost 1% of its original rotational speed. Assuming that all mechanical energy losses are due to kinetic friction between the plate and floor, find that coefficient of kinetic friction. (Ignore the slow speed of the plate’s sliding.)

h. A rigid, massless rod connects three point masses as shown here. Doing a total of 192 J of work on this object (which is initially at rest in your hand), you throw it across a level field, launching it so that its center of mass leaves your hand at an angle of 34.0° above the horizontal. At the peak of its arc as a projectile, the object’s center of mass has gained 7.00 m in altitude (i.e. above its altitude when released).

(i) Assuming that this projectile maintained its rotational speed throughout the upward portion of its arc, what was that \( \omega \) value?

Now suppose that, during the downward portion of the object’s arc, \( m_2 \) slides 10.0 cm toward \( m_3 \).

(ii) What is the rotational speed now— with \( m_2 \) in its new position?

(iii) And now what is the speed of \( m_j \) as measured from the object’s axis of rotation (i.e. what is \( v_{r,j} \) around the axis of rotation)?

Ignore any effects of air drag and wind, and assume a constant local \( g \) value of 9.80 m/s².
i. A certain baton consists of a thin, rigid, massless rod, 1.00 m long with a different point mass attached to each end ($m_1 = 1.00 \text{ kg}; m_2 = 4.00 \text{ kg}$). You may assume normal local $g_{\text{earth}}$.

(i) Initially, the rod is pinned at its center to a fulcrum supported by level ground. A force $F$ is applied as shown, at a point midway between $m_1$ and the fulcrum, so that the rod stays horizontal and in static mechanical equilibrium. Find the magnitude of $F$.

(ii) Now $F$ is removed, and the baton is removed from fulcrum. It is then hurled into the air so that it rotates freely at 300 rpm. Find its rotational kinetic energy.
5. j. Rigid, massless rods connect three point masses as shown in this overhead view. This object is set at rest on level, frictionless ice, then pinned at \( m_2 \) to that ice so that it can rotate freely (without friction, in a horizontal plane) around \( m_2 \). Then a torque of 87.0 N·m is applied to the object for just 4.00 s (at which time the torque ceases). Ignore wind/air drag.

(i) At what translational speed does the object’s center of mass now move?

(ii) If the pin could then somehow pull in (shorten—like an antenna or telescope) the 3-m rod so that its new length was 1.50 m, how much work would need to be done by the pin to accomplish this?
5. k. A long thin, rigid, uniform rod of mass $M$ and length $L$ has one end attached at ground level to a frictionless pivot. The rod is initially upright and at rest, as shown. Then it falls freely (with no push) onto level ground, as shown.

You may consider $M$, $L$, and local $g$ to be known values.

Evaluate (T/F/N) each statement. Justify your answers fully with any valid mix of words, drawings and calculations.

(i) During its fall, the rod’s center of mass has the same angular speed, $\omega$, as its free end.

(ii) When the rod hits the ground, the impact speed ($v_{\text{impact.end}}$) of the free end is: $v_{\text{impact.end}} = \sqrt{2gL}$
5. 1. A rigid beam (mass = 400 kg; length = 12.0 m) has a pulley
(mass = 30.0 kg; outer radius = 1.50 m) mounted at its upper end; and the beam’s lower end is anchored with a pin to a wall. The beam is also supported by a horizontal tension cable connected to the wall, as shown here. The distance along the beam from the pulley axle to the cable attach point is 3.50 m.

A strong, thin cord has been wrapped several times around the pulley and then connected to a block \( (m_3 = 100 \text{ kg}) \)—which hangs at rest so long as the pulley is secured to the beam with a removable “brake” pin to prevent it from rotating.

(i) Find the tension in the cable when everything is at rest.

(ii) Now suppose the brake pin is suddenly removed. When the block has lowered by a vertical distance of 2.50 m, how fast is it moving?

Ignore air drag and any friction in the anchor pin, cable attachment, or pulley axle. Assume that the beam is a solid, uniform rod; the pulley is a solid, uniform disk; and the cord is massless and never slips with respect to the pulley rim.
6. a. A solid, uniform sphere of mass 2.0 kg and radius 1.7 m starts from rest and rolls without slipping down an inclined plane of height 5 m. What is the angular velocity of the sphere at the bottom of the inclined plane?

b. A bowling ball rolls without slipping, first along a level track, then up a ramp onto another level section of the track, gaining 0.340 m in altitude. If its translational speed along the lower track level was 3.15 m/s, find its translational speed at the top.

c. A basketball of mass \( m \) and radius \( r \) rolls without slipping up a hill. The angle between the hill and the horizontal is \( \theta \). The initial speed of the basketball is \( v \). The magnitude of the acceleration due to gravity is \( g \). Expressed in terms of \( m, r, \theta, v \) and/or \( g \), what vertical height, \( h \), does the ball gain before it comes to rest (momentarily)? Assume that the ball is a rigid body, and that friction and air resistance can be ignored.
6. **d.** The rear brakes on a bicycle consist of a pair of rubber pads that clamp down and rub (μk = 0.863) on the rim of the rear wheel in order to slow it. Suppose the bike is rolling (without slipping) along a straight, level road at 9.25 m/s when the rider applies the rear brakes. If the normal force applied by each brake pad on the wheel rim is 74.1 N, find the speed of bike and rider when the wheels have revolved exactly six times after braking began. Ignore air and rolling resistance, and assume that the bike has two identical wheels; and that the brake pads apply their forces at the wheel’s outer radius.

\[ I_{\text{wheel}} = 0.268 \text{ kg \cdot m}^2 \quad r_{\text{wheel}} = 0.345 \text{ m} \quad m_{\text{bike\&rider}} = 107 \text{ kg} \]
6. e. In two separate trials, a solid sphere (A) and a hollow spherical shell (B) are each released from rest and allowed to roll for the same distance down the same slope (shown here). Evaluate (T/F/N) each statement. Justify your answers fully with any valid mix of words, drawings and calculations.

(i) Their final translational speeds compare like this: \( v_{fB} \approx (1.09)v_{fA} \)

(ii) At any given moment during its motion along the slope, the solid sphere has about 28.6\% of its total kinetic energy in the form of \( K_R \).

(iii) Their average translational acceleration magnitudes compare like this: \( a_B \approx (0.917)a_A \)
6. f. Beginning from rest, a wheel of mass 20.0 kg and outer radius 0.750 m rolls without slipping down a slope onto level ground, losing a total of 4.30 m in altitude. When it arrives on level ground, its total kinetic energy is 75% $K_T$ and 25% $K_R$.

Assume that the wheel’s center of mass is located at its geometric center (and this is the axis around which it turns as it rolls), but do not assume that the wheel is a uniform disk. Also, assume normal local $g_{\text{earth}}$, and ignore air resistance and rolling friction.

(i) How fast does the wheel’s axle travel over the level ground?

(ii) What is the moment of inertia of this wheel (about its center of mass)?
6. g. A solid sphere and a solid cube have equal masses.
You test each, one at a time, on the slope shown here.
In each test, you release the object from rest at point A.
The sphere rolls without slipping, but the cube slides.
In this particular case, it so happens that each object
reaches point B with the same translational speed.
Find the coefficient of kinetic friction between the cube and the slope.
6. h. An ideal spring (stiffness = 910 N/m) with a frictionless “bumper,” as shown, is attached along a level surface to a wall and compressed by a distance of 0.320 from its relaxed equilibrium position. A ball (a solid, uniform sphere, mass = 3.68 kg, radius = 0.465 m) is placed in front of the bumper. When the spring is released, the ball slips (“skids”—i.e. it doesn’t roll) horizontally for a distance of 0.791 m. Then it rolls without slipping thereafter, up the ramp and off the end, which is 0.879 m above the level surface. The coefficient of kinetic friction for the entire surface and ramp is 0.245. Find the rotational speed of the ball as it leaves the ramp. Neglect air resistance.