

Dispersion Relation for Waves on a String

1. In a finite string, certain waveforms satisfy Newton's law, and only a few have the correct wavelengths to fit the constraints imposed by the boundaries ("boundary conditions").
2. For any system in which waves propagate, there exists a well-defined relationship between ω and k - the "dispersion relation".

This exercise is one that you may have studied in introductory physics courses. The purpose here is to discuss this simple, but very rich example in more sophisticated language than you would have used in previous courses to set the stage for more complex examples to come later in the course and in the major. Aspects of this worksheet may appear in your homework assignment, in which case you must turn in that work. This worksheet does not need to be turned in separately.

(SEE OVER FOR A TABLE TO RECORD YOUR RESULTS)

- Measure the frequencies of two normal modes of oscillation of a rope (different groups are assigned different modes).
- Plot and describe the dispersion relation - the relationship between ω and k . Excel is quickest; we'll do this as a group.
- Calculate the velocity of wave propagation from the dispersion relation plot (hwk).
- Calculate the velocity of propagation from the mass density and tension of the string and comment on whether the model we used to generate this calculation is appropriate (hwk).
- Discussion: If you were to make a superposition of waves of different frequencies, what shape would the resulting waveform have, and would can you say about the various velocities we have discussed? (hwk)
- Understand how the waveform you observe is a solution to Newton's equation $F = ma$ for this system.
- Theory: Add two (single-frequency) traveling waves propagating in opposite directions and establish that a standing wave results. (hwk)
- Food for thought: later in this course and in future courses, you will encounter systems in which waves propagate, and the relationship between the frequency and the wave vector k is very different from this "non-dispersive" example.

NORMAL MODES OF A STRING FIXED AT BOTH ENDS:

Record the "mode" as the number of nodes EXCLUDING the fixed ends.

| Mode | Frequency (Hz) | Ang. frequency | Wavelength | Wave vector |
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Other useful system parameters:

Length of the string:

Mass density of the string:

Tension in the string: