## Name:

$\qquad$

## Number of Paths

Working in small groups (2 or 3 people), solve as many of the problems below as possible. Try to resolve questions within the group before asking for help. Each group member should then write up solutions in their own words.

The Sign of Work: For the force vector shown at left, draw small displacement vectors $\overrightarrow{d u}, \overrightarrow{d v}$, and $\overrightarrow{d w}$ such that the work done by $\vec{F}$ is positive, zero, and negative for small displacements in those directions, respectively.
This gets at the dot product relationship students will need to perform the vector path integral. It does not get at the ideas that you have to chop the whole path into small pieces to do the integral


Prep - Change the notation for the differential displacement vectors as is appropriate for your class.
Follow-up - "Is there only one correct answer for each?" Students usually draw the parallel/antiparallel/perpendicular displacements and don't consider other options.
Follow-up - Does the sign of the charge on the particle change your answer? Juniors tend to know that test charges are always positive and so only consider positive

Number of Paths: Points are marked on fields $\vec{G}$ and $\vec{H}$. How many paths can you draw between each pair of points where the integral is positive, negative, or zero?

| Number of paths with |  |  | Starting | Ending | Number of paths with |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\int \vec{G} \cdot d \vec{r}>0$ | $\int \vec{G} \cdot d \vec{r}=0$ | $\int \vec{G} \cdot d \vec{r}<0$ | Point | Point | $\int \vec{H} \cdot d \vec{r}>0$ | $\int \vec{H} \cdot d \vec{r}=0$ | $\int \vec{H} \cdot d \vec{r}<0$ |
|  |  |  | $\Delta$ |  |  |  |  |
|  |  |  |  | $\star$ |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  | $\star$ | $\boldsymbol{\Delta}$ |  |  |  |

Student Discussions: Students tend to count "One, two, infinity"
Whole Class Discussion: Discuss what paths are the most convenient for estimating the sign of the path integral and why
Match a Surface: One of the vector fields corresponds to (part) of your surface. Where does it match, and how do you know?
Possible Reasoning: The gradient is perpendicular to the level curves and points in the direction of increasing function. The line integral corresponds to a change in height on the surface.
Student Discussions" Some students need to be told that the vector field is the gradient. The gradient will point in the direction of increase. Note that if the field is the electric field and the surface is the potential, there is a relative minus sign so that the field points in the direction of
decreasing potential. This ideas is brought out much more strongly in the "Work and Electric Field" activity.

Other Surface? Could the other vector field correspond to a surface? Explain why or why not. The other vector field is a magnetic field due to a infinite straight linear current and is not conservative - so no corresponding surface .

# Activity Evaluation <br> What was the main point of this activity? 

Describe one thing you understand as a result of this activity.

Describe one thing that is confusing after completing this activity.


Figure 1. Field $\vec{G}$


Figure 2. Field $\vec{H}$

