## Probabilities of S.G. Analyzers(Spin-1/2)

Set up an experiment to measure the spin projection  $S_z$  along the z-axis twice in succession as shown below. You need an extra analyzer and another counter (see the SPINS notex for help). Run the experiment and note the results. Focus your attention on the second analyzer. The input state is denoted  $|+\rangle$  and there are two possible output states  $|+\rangle$  and  $|-\rangle$ . What is the probability that an atom entering the second analyzer (state  $|in\rangle = |+\rangle$ ) exits the spin up port (state  $|out\rangle = |+\rangle$ ) of the second analyzer? This probability is denoted in general as  $P(out) = ||\langle out|in\rangle|^2$ , and in this specific case as  $P(+) = ||\langle out|in\rangle|^2 =$  $||\langle +|+\rangle|^2$ . What is the probability of exiting the spin down port (state  $|-\rangle$ )? What conclusions can you draw from the



measurements performed in this experiment?

Now, still using the same apparatus as above, change the orientation directions of the analyzers. You can choose directions X, Y, or Z, which are oriented along the usual xyz-axes of a Cartesian coordinate system (ignore the fourth direction  $\hat{n}$  for now). When a direction other than Z is chosen, we use a subscript to distinguish the output states (e.g.,  $|-\rangle_x$ ). If we allow ourselves to also use the spin down port of the first analyzer as input to the second analyzer (not both up and down at the same time), then there are six possible input states and six possible output states for the second analyzer, which are listed in the table below. Your task is to measure the probabilities  $P(out) = ||\langle out|in \rangle|^2$ corresponding to these input and output states. Remember that this is the probability that an atom leaving the first analyzer also makes it through the second analyzer to the appropriate detector, and not the total probability for getting from the oven to the detector. The experiment performed in # 3 above (with both analyzers along the z-axis) gave the result  $||\langle +|+\rangle|^2 = 1$ , which is already entered in the table. Now do all other possible combinations and fill in the rest of the table.

Unknown  $|\psi_1\rangle$ 

$\left \left \left\langle out in ight angle  ight ^{2}$	$ +\rangle$	$ -\rangle$	$ + angle_x$	$ - angle_x$	$ + angle_y$	$ - angle_y$
(+)	1					
(-)						
$\langle + _x$						
$\langle - _x$						
$\langle + _y$						
$\langle - _y$						