Working in groups of two or three, do the following problem.

1. **COSMIC RAYS I**

Consider μ-mesons produced by the collision of cosmic rays with gas nuclei in the atmosphere 60 kilometers above the surface of the earth, which then move vertically downward at nearly the speed of light. The halflife before μ-mesons decay into other particles is 1.5 microseconds (1.5×10^{-6} s).

Assume all the mesons are traveling straight down at the same speed.

(a) Assuming it doesn't decay, how long would it take a μ-meson to reach the surface of the earth?

(b) Assuming there were no time dilation, approximately what fraction of the mesons would reach the earth without decaying?

(c) In actual fact, roughly $\frac{1}{8}$ of the mesons reach the earth! How fast are they going?

Working in groups of two or three, consider one or more of the following three problems.

2. **FAST ELECTRONS**

The Stanford Linear Accelerator (SLAC) accelerates electrons to a final *kinetic* energy of 40 GeV along a straight path approximately 3000 meters long. (1 GeV $= 10^3$ MeV $= 10^9$ eV)

(a) Assume that the electrons increase their kinetic energy by approximately equal amounts for every meter traveled in the accelerator. How much kinetic energy do they gain per meter?

(b) Using *Newtonian* mechanics, how far would an electron have to travel in the accelerator before its speed were equal to the speed of light?

The rest mass m of an electron satisfies $mc^2 \approx 0.5$ MeV.

(c) Using special relativity, how fast are the electrons traveling when they emerge from the end of the accelerator?

You may assume that $1 - \frac{v}{c} \ll 1$.

3. **MASS ISN'T CONSERVED**

Two identical lumps of clay of (rest) mass m collide head on, with each moving at $\frac{3}{5}c$. What is the mass of the resulting lump of clay?

4. **COLLISIONS I**

Consider the head on collision of 2 identical particles each of mass m and energy E.

(a) In Newtonian mechanics, what multiple of E is the energy E' of one particle as observed in the reference frame of the other?

(b) In special relativity, what is the energy E' of one particle as observed in the reference frame of the other?

(c) Suppose we collide 2 protons ($mc^2 = 1$ GeV) with energy $E = 30$ GeV. Roughly what multiple of E is E' in this case?

Additional problems to think about.

5. **COSMIC RAYS II**

At least 1 cosmic-ray particle has been observed with an estimated energy of 10^{20} eV.

(a) Conservatively assuming that this particle was as heavy as a proton ($mc^2 \approx 1$ GeV), how long would it take to cross our galaxy *as measured by its own clock*?

The diameter of the galaxy is about 10^5 *light years; 1 light year is roughly* 10^{16} *meters;* 1 *year* $\approx 3 \times 10^7$ *seconds.*

(b) How much energy would a particle of mass m have if it were going fast enough so that our entire galaxy would appear to be Lorentz contracted to the size of a proton?

The diameter of a proton is roughly 1 fermi, which is 10^{-15} *meters.*

(c) How much mass would have to be converted to energy to give a proton the desired speed?

In conventional units, the mass of a proton is roughly 1.67×10^{-27} *kilograms.*

6. **SYMMETRIC SCATTERING**

Consider the elastic scattering of 2 identical particles, in which 1 particle is initially at rest. After the collision, the velocity of each particle makes a certain angle with the original direction of motion; assume that these angles are equal. (The total scattering angle θ is thus equal to twice this angle.)

(a) Working in the lab frame (in which one particle was initially at rest), compute the scattering angle.

(b) Working in the center-of-mass frame (in which the 2 particles initially have equal and opposite velocities), compute the scattering angle.

(c) Show that in the limit of small velocities the result reduces to the Newtonian result, namely $\theta = \frac{\pi}{2}$.