Student Responses to Chain Rule Problems in Thermodynamics

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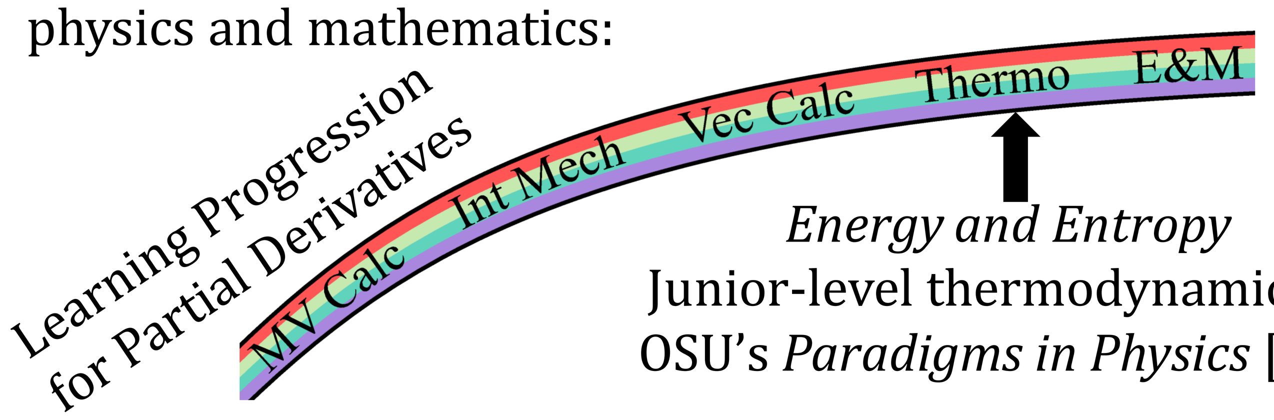
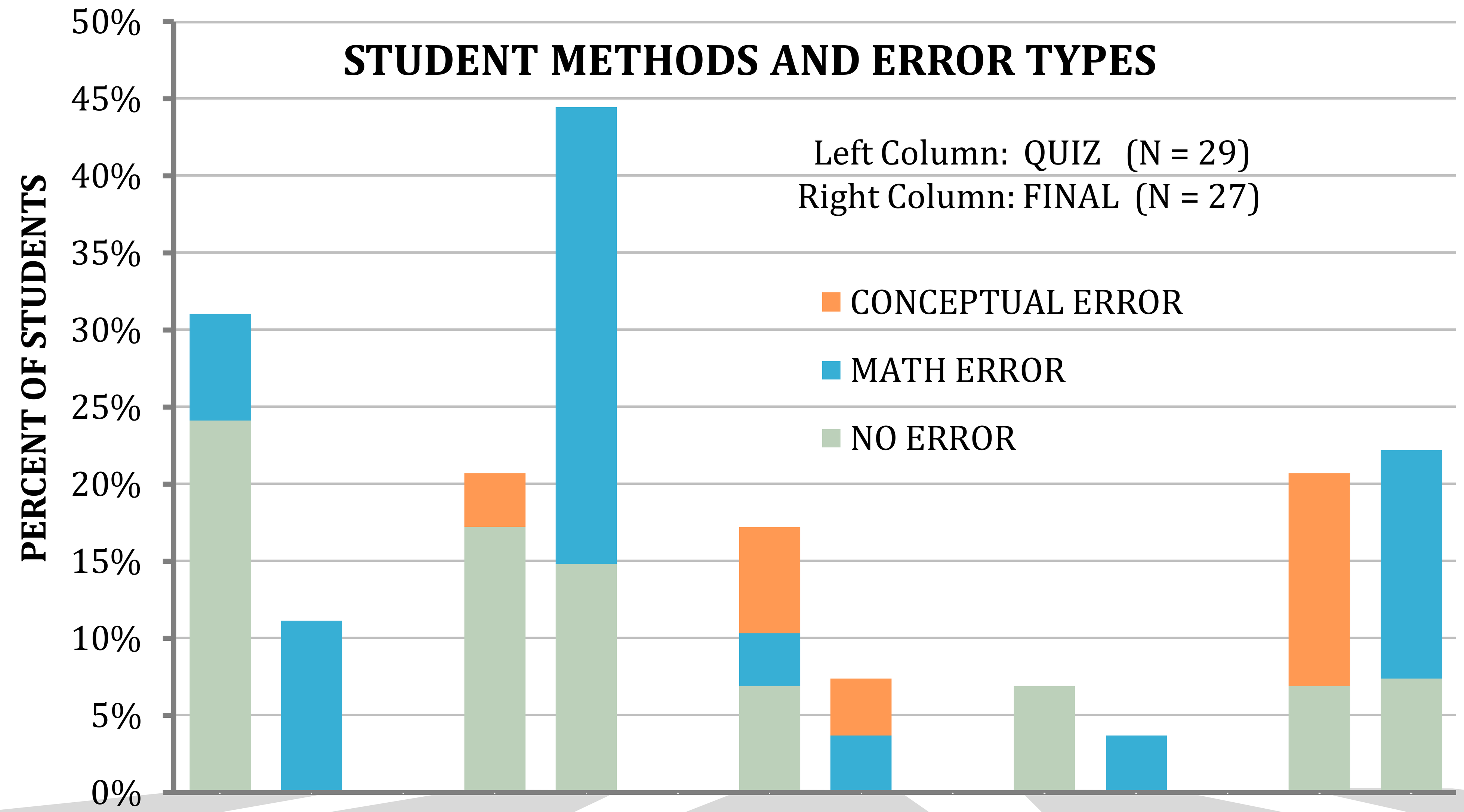
INTRODUCTION

What methods do students use to evaluate partial derivatives that "require" a multivariable chain rule?

- We studied student responses to 2 chain rule problems:
  - Quiz prompt (no thermodynamics context)
  - Final prompt (implicit thermodynamics context)
- We identified 5 solution methods using emergent coding.
- Student errors were also identified and categorized.
- Part of an expansive study [1] to describe a broad learning progression for partial derivatives across advanced courses in physics and mathematics:



Energy and Entropy
Junior-level thermodynamics in OSU's *Paradigms in Physics* [2-3]



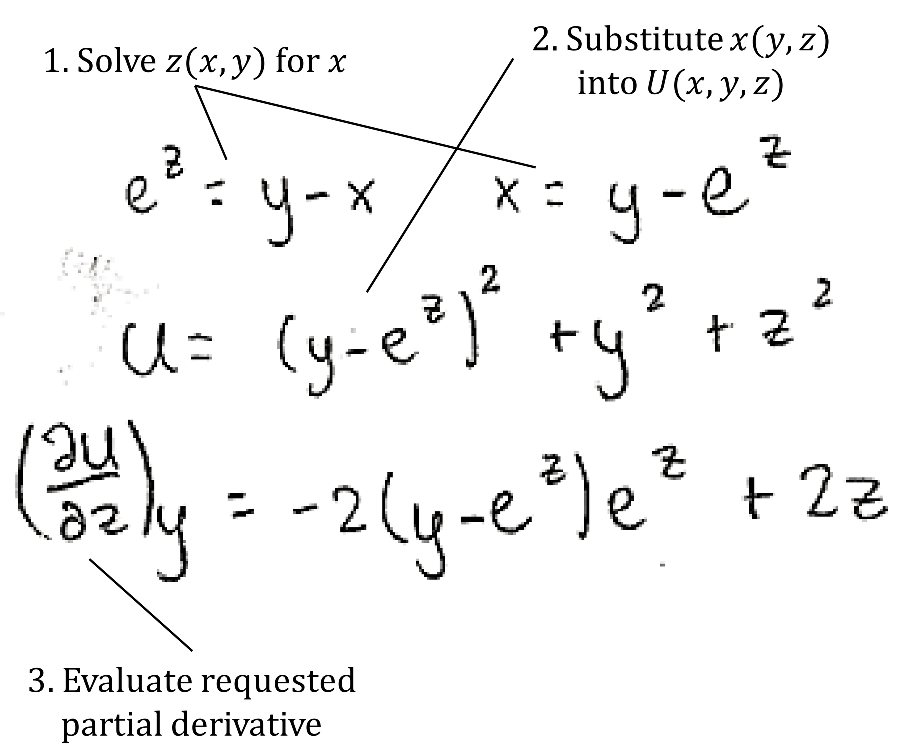
THE PROMPTS

Given the definitions, evaluate the requested partial derivative.

QUIZ PROMPT	FINAL PROMPT
$U = x^2 + y^2 + z^2$	$S = Nk_B\left(\ln\left(\frac{N - Vb}{NC}T^{\frac{3}{2}}\right) + \frac{5}{2}\right)$
$z = \ln(y - x)$	$U = \frac{3}{2}Nk_BT - \frac{aN^2}{V}$
Find $\left(\frac{\partial U}{\partial z}\right)_y$	Find $\left(\frac{\partial U}{\partial V}\right)_S$
(left column in table) Assigned on last class day – Friday	(right column in table) Assigned on final exam – Monday

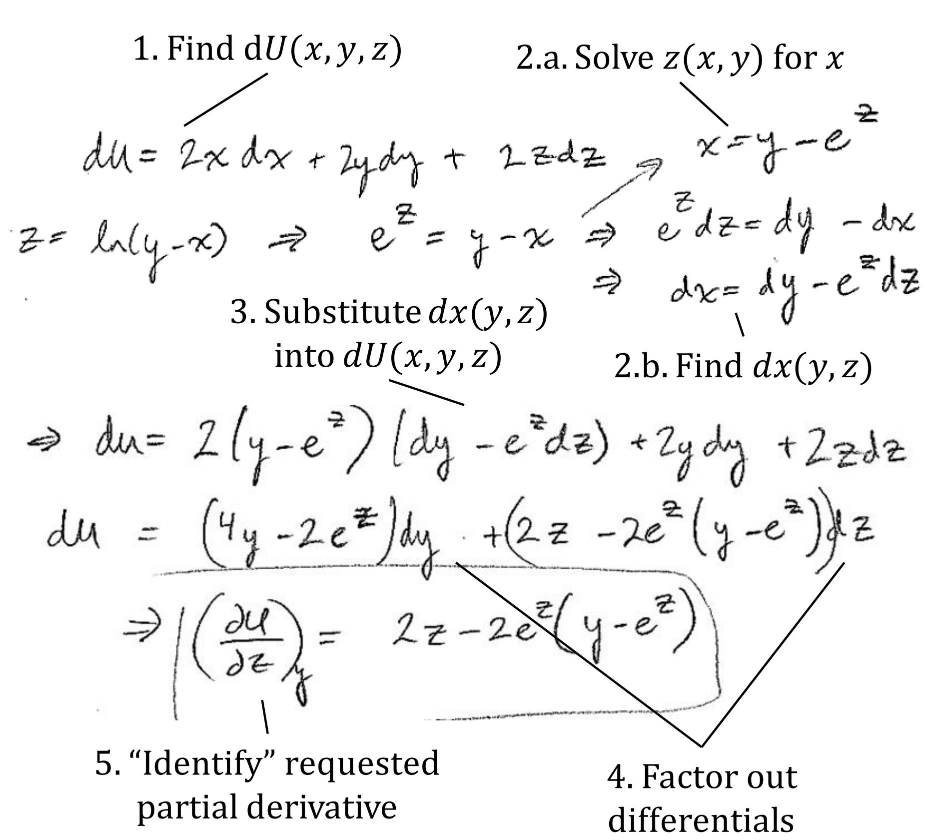
VARIABLE SUBSTITUTION

1. Solve $z(x, y)$ for x
2. Substitute $x(y, z)$ into $U(x, y, z)$
3. Evaluate requested partial derivative



DIFFERENTIAL SUBSTITUTION

1. Find $dU(x, y, z)$
2. Solve $z(x, y)$ for x, find $dx(y, z)$
3. Substitute $dx(y, z)$ into $dU(x, y, z)$
4. Factor out differentials
5. "Identify" partial derivative



IMPLICIT DIFFERENTIATION

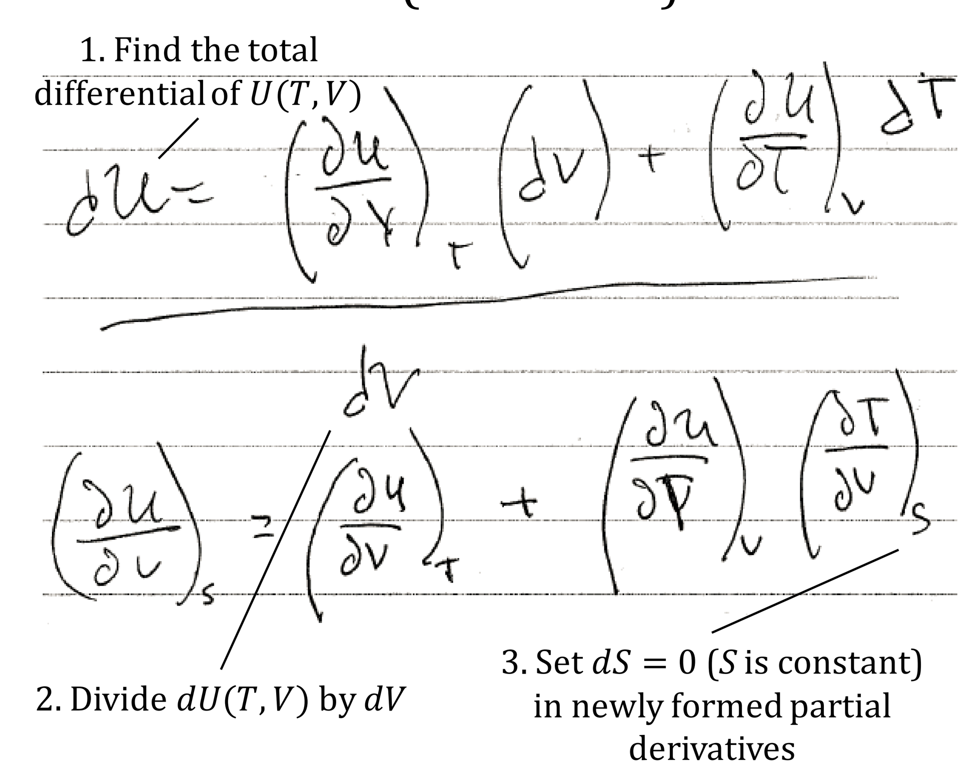
1. Without intermediate written work, write the chain rule for the requested partial derivative
2. Evaluate the chain rule's partial derivatives (not shown)

1. Without intermediate written work, write the chain rule (for the requested partial derivative).

$$\left(\frac{\partial U}{\partial z}\right)_y = \left(\frac{\partial U}{\partial x}\right)_{y,z}\left(\frac{\partial x}{\partial z}\right)_y + \left(\frac{\partial U}{\partial z}\right)_{x,y}$$

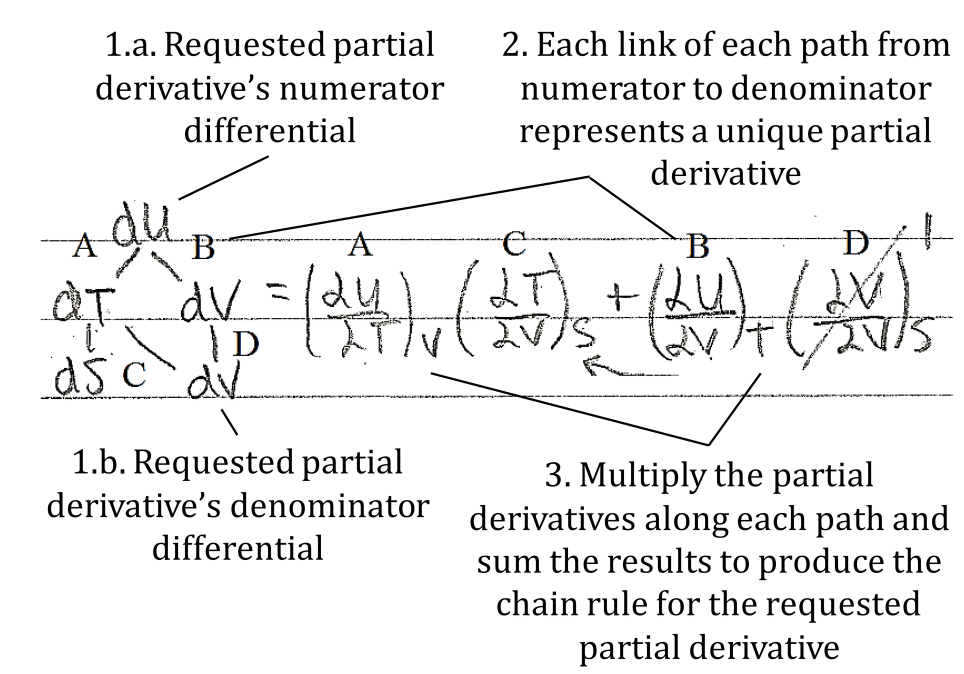
DIFFERENTIAL DIVISION

1. Find $dU(T, V)$
2. Divide $dU(T, V)$ by dV
3. Set $dS = 0$ (S is constant) in newly formed partial derivatives
4. Evaluate the chain rule's partial derivatives (not shown)



CHAIN RULE DIAGRAM

1. Build chain rule tree diagram for the requested partial derivative
2. Associate all branches with a partial derivative
3. Construct the chain rule by following all paths from dU to dV
4. Evaluate the chain rule's partial derivatives (not shown)



ERROR TYPE	ERROR DESCRIPTION	ERROR CONCLUSIONS
Conceptual Error	Errors during manipulation of derivatives, differentials, or chain rule diagrams. These errors provide insight into students' lack of conceptual understanding of the solution methods.	• Students often "mis-identified" partial derivatives from total differentials. • Students often did not correctly hold variables constant while evaluating partial derivatives. • Several students mis-built or mis-read chain rule diagrams on the Quiz.
Math Error	Algebraic errors, sign errors, inadvertently dropped terms, *etc.*	• Math errors were prevalent on the Final prompt but conceptual errors were not.
No Error	No errors were made.	• Every student who made no error on the Final used differential substitution or chain rule diagrams.

METHOD CONCLUSIONS

- Students used a variety of solution methods but favored different methods on the Quiz than on the Final.
- Variable substitution was less common on the Final, where the algebra is more difficult.
- Differential substitution was more common on the Final.
- Students who used a chain rule diagram improved from Quiz to Final.

REFERENCES

[1] See Manogue's poster, number
[2] C. A. Manogue *et al.*, Am. J. Phys., **69** (2001)
[3] C. A. Manogue *et al.*, Phys. Today, **56** (2003)

ACKNOWLEDGEMENTS

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