Student Interpretations of Partial Derivatives

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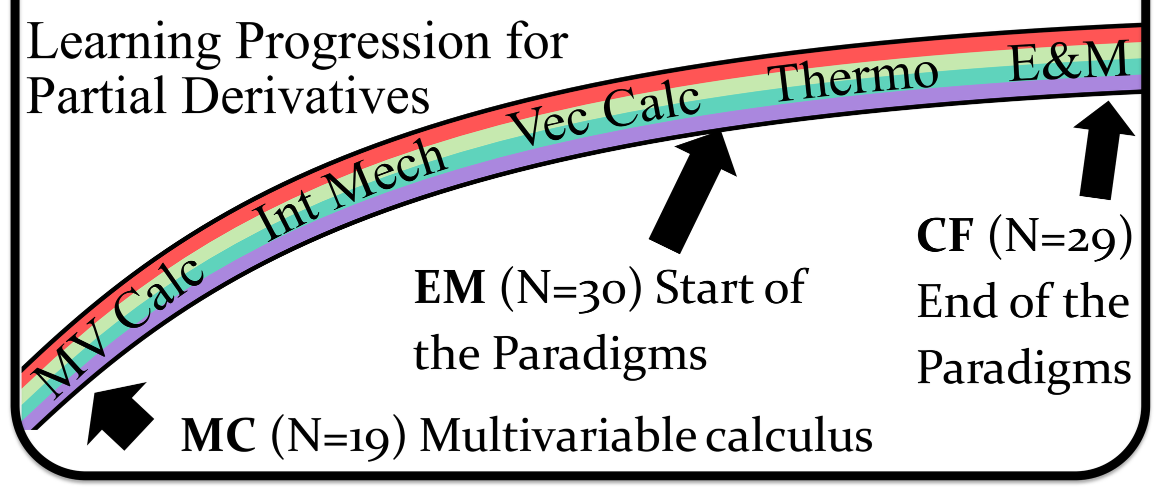
Background

We are developing a learning progression [1] for partial derivatives; one aspect is taking research "snapshots" at different points along the progression, such as:

- **How do student ideas about derivatives and partial derivatives evolve as they progress through undergraduate math and physics?**

(3-question survey given in three courses)



Question 1

For a function h, the derivative of h with respect to x is sometimes known as a *rate of change*. Explain the meaning of rate of change in this context.

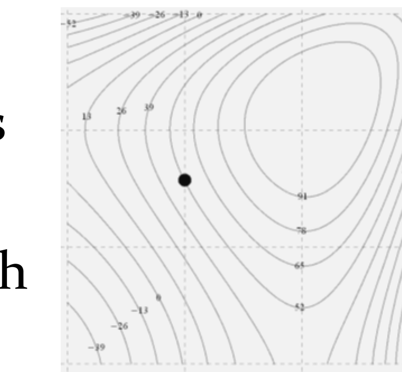
Question 2

During a previous class period, you were provided with a plastic surface, which is a representation of a function $h(x, y)$. Explain the meaning of the *derivative* of h with respect to x in this context.

[2]

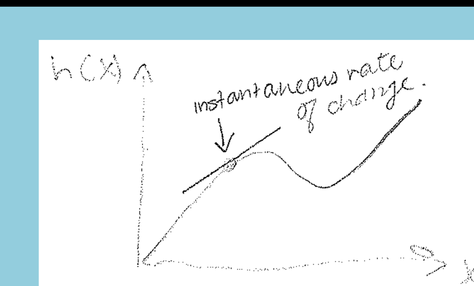
Question 3

The contour graph also represents a function $h(x, y)$. Explain the meaning of the *derivative* of h with respect to x in this context.



Slope

"Rate of change is the slope of the function at any point along line h."



	Q1	Q2	Q3
MC	53%	42%	42%
EM	17%	27%	17%
CF	28%	20%	20%

- More prevalent among math than physics students.
- Only about half of these students drew a graph in support of their answer.
- Few drew or discussed a tangent line.

Change

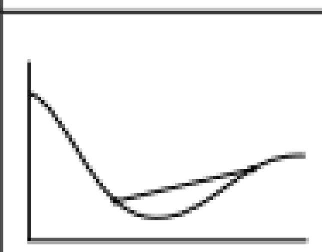
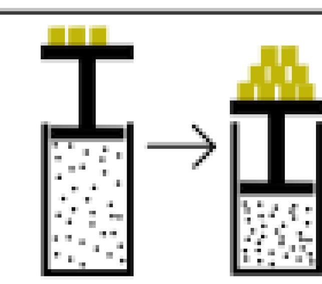
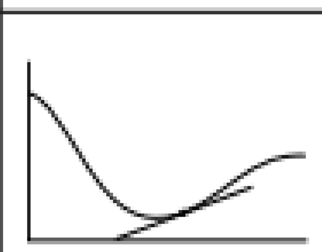
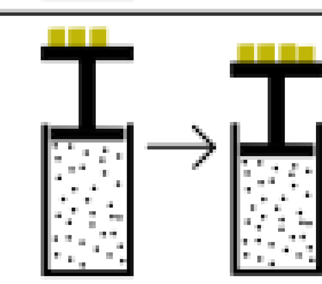
"The rate of change expresses how much h changes as x changes."

"The derivative of a function captures the instantaneous change of the function."

	Q1	Q2	Q3
MC	37%	37%	37%
EM	70%	60%	47%
CF	82%	60%	66%

- More prevalent among physics than math students.
- Students commonly used the phrase "how much" the function changes across all questions.
- Few students discussed the derivative as a ratio.

Theoretical perspective – Concept image [3-4]

Process-object layer	Graphical	Verbal	Symbolic	Numerical	Physical
	Slope	Rate of Change	Difference Quotient	Ratio of Changes	Measurement
Ratio		"average rate of change"	$\frac{f(x+\Delta x)-f(x)}{\Delta x}$	$\frac{y_2-y_1}{x_2-x_1}$ numerically	
Limit		"instantaneous …"	$\lim_{\Delta x\to 0}\cdots$	...with Δx small	
Function		"... at any point/time"	$f'(x)=\cdots$	... depends on x	tedious repetition

What to hold constant

Hold y constant

"How does h change as x is wiggled and y is constant."

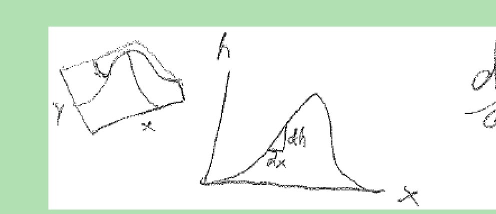
Only x change

"How much the distance between contour lines changes as x (and only x) is changed by a given amount."

In a direction

"How quickly and in what direction the value of h would change if you moved only in the x-direction."

Cross-section



- Common language like "hold y constant" was prevalent, especially on Question 2.
- Students who talked about moving in a direction may be thinking graphically, but rarely drew graphs.
- A surprising number of physics students did not discuss this idea at all!

	Question 2			Question 3		
	MC	EM	CF	MC	EM	CF
Hold y constant	16%	33%	41%	5%	3%	24%
Only change x	0%	7%	7%	11%	3%	7%
In a direction	32%	43%	38%	11%	53%	48%
Cross-section	11%	3%	3%	16%	10%	0%
Did not discuss	26%	17%	21%	47%	20%	10%

Conclusions from comparing student work to theoretical perspective

- Student interpretations tended to be verbal or symbolic in nature.
- Few students discussed the derivative as a *ratio* – many students wrote dh/dx, but this seemed to be common notation rather than division.
- Almost no students discussed the derivative as an explicit limit – but 30-40% of physics students did use words like "small" or "instantaneous."
- Few used language that distinguished between the derivative at a point *vs.* as a function.

Our results are only indicative of students' first-level interpretations – further research is necessary to explore the concept image of these student populations.

Conclusions from open-coding analysis of student work

- Math students favor *slope* while physics students favor *change*.
- Students' interpretations at the end of a year of junior physics are mostly the same as at the start of the year.
- The "how much h changes" language was incredibly common (~50% of physics students); while it is not present in physics or calculus textbooks, it may be common language among physics experts.
- Students were not more likely to interpret the derivative graphically in graphical contexts.
- Discussions of what to hold constant were somewhat different given the two graphical contexts, but student language tended to be symbolic rather than graphical.

Refs: [1] See Manogue's poster, number
[2] raisingcalculus.winona.edu
[3] M.J. Zandieh, CBMS Issues in Math. Ed. **8** (2000).
[4] D. Roundy *et al.*, in RUME Conf. Proceedings (2015).

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