Homework for Quantum Calculations on the Ring I (Comparing Different Representations)

Before you begin, recall that an arbitrary state $|\Phi\rangle$ can be written in the L_z eigenbasis as

$$|\Phi\rangle \doteq \begin{pmatrix} \vdots \\ \langle 2|\Phi\rangle \\ \langle 1|\Phi\rangle \\ \langle 0|\Phi\rangle \\ \langle -1|\Phi\rangle \\ \langle -2|\Phi\rangle \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ a_2 \\ a_1 \\ a_0 \\ a_{-1} \\ a_{-2} \\ \vdots \end{pmatrix}$$

For this question, you will carry out calculaions on each of the following normalized quantum states on a ring:

$$|\Phi_a\rangle = \sqrt{\frac{4}{15}}\,|4\rangle + \sqrt{\frac{1}{15}}\,|2\rangle + \sqrt{\frac{4}{15}}\,|1\rangle + \sqrt{\frac{3}{15}}\,|0\rangle + \sqrt{\frac{1}{15}}\,|{-3}\rangle + \sqrt{\frac{2}{15}}\,|{-4}\rangle$$

$$|\Phi_b\rangle \doteq \begin{pmatrix} \vdots \\ \sqrt{\frac{4}{15}} \\ 0 \\ \sqrt{\frac{1}{15}} \\ \sqrt{\frac{4}{15}} \\ \sqrt{\frac{3}{15}} \\ 0 \\ 0 \\ \sqrt{\frac{1}{15}} \\ \sqrt{\frac{2}{15}} \\ \vdots \end{pmatrix}$$

$$\Phi_c(\phi) = \sqrt{\frac{1}{30\pi}}\left(\sqrt{4}\left(e^{i4\phi} + e^{i\phi}\right) + \sqrt{3} + \sqrt{2}e^{-i4\phi} + e^{i2\phi} + e^{-i3\phi}\right)$$

For each question state the postulate(s) of quantum mechanics you use to complete the calculation and show explicitly how you use the postulates to answer the question.

1) If you measured the z-component of angular momentum for each state, what is the probability that you would obtain $4\hbar$? 0? $-2\hbar$?

2) If you measured the energy for each state, what is the probability that you would obtain 0? $\frac{\hbar^2}{2I}$? $\frac{16\hbar^2}{2I}$? $\frac{25\hbar^2}{2I}$?

3) How are the calculations you made for the different state representations similar and different? In a short paragraph, compare and contrast the calculation methods you used for each of the different representations (ket, matrix, wavefunction).

4) If you measured the z-component of angular momentum, what other possible values could you obtain with non-zero probability?

5) If you measured the energy, what other possible values could you obtain with non-zero probability?

by Corinne Manogue and Kerry Browne

Homework for Quantum Calculations on a Ring II (Time Dependence)

In this problem, you will carry out calculations on the following normalized abstract quantum state on a ring:

$$|\Psi\rangle = \sqrt{\frac{1}{4}}\left(|1\rangle + \sqrt{2}\,|2\rangle + |3\rangle\right)$$

1) You carry out a measurement to determine the energy of the particle at time t=0. Calculate the probability that you measure the energy to be $\frac{4\hbar^2}{2I}$.

2) You carry out a measurement to determine the z-component of the angular momentum of the particle at time t=0. Calculate the probability that you measure the z-component of the angular momentum to be $3\hbar$.

3) You carry out a measurement on the location of the particle at time, t=0. Calculate the probability that the particle can be found in the region $0 < \phi < \frac{\pi}{2}$.

4) You carry out a measurement to determine the energy of the particle at time $t = \frac{2I}{\hbar^2}\frac{\pi}{4}$. Calculate the probability that you measure the energy to be $\frac{4\hbar^2}{2I}$.

5) You carry out a measurement to determine the z-component of the angular momentum of the particle at time $t = \frac{2I}{\hbar^2}\frac{\pi}{4}$. Calculate the probability that you measure the z-component of the angular momentum to be $3\hbar$.

6) You carry out a measurement on the location of the particle at time, $t = \frac{2I}{\hbar^2}\frac{\pi}{4}$. Calculate the probability that the particle can be found in the region $0 < \phi < \frac{\pi}{2}$.

7) Write a short paragraph explaining what representation/basis you used for each of the calculations above and why you chose to use that representation/basis.

8) In the above calculations, you should have found some of the quantities to be time dependent and others to be time independent. Briefly explain why this is so. That is, for a time dependent state like $|\Psi\rangle$ explain what makes some observables time dependent and others time independent.

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Homework for Quantum Calculations on a Ring III

Consider the following normalized quantum state on a ring:

$$\Phi(\phi) = \sqrt{\frac{8}{3\pi}} \sin\left(3\,\phi\right)^2 \cos\left(\phi\right)$$

1) If you measured the z-component of angular momentum, what is the probability that you would obtain $\hbar$? $-3\hbar$? $-7\hbar$?

2) If you measured the z-component of angular momentum, what other possible values could you obtain with non-zero probability?

3) If you measured the energy, what is the probability that you would obtain $\frac{\hbar^2}{2I}$? $\frac{4\hbar^2}{2I}$? $\frac{25\hbar^2}{2I}$?

4) If you measured the energy, what possible values could you obtain with non-zero probability?

5) What is the probability that the particle can be found in the region $0 < \phi < \frac{\pi}{4}$? In the region $\frac{\pi}{4} < \phi < \frac{3\pi}{4}$?

by Corinne Manogue