Plastic vs. Solid Resources In Upper-division E&M

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Abstract: The ability of juniors in physics at Oregon State University to solve for the magnetic vector potential of a spinning ring of charge is analyzed using Sayre and Wittmann's model of resource plasticity. None of the 17 students, working in groups of two or three, was able to solve the problem in the given time. Those students who used geometric reasoning to connect to something they solidly understood did not accept incorrect answers and knew they had not yet achieved a solution, whereas many other students were willing to connect their answers to more tenuously understood "plastic resources" and their answers contained errors. We discuss these results and their instructional implications concerning the degree to which students are encouraged to connect to solid resources as they solve problems, compared to being encouraged to consider connection to plastic resources as being "good enough".

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INTRODUCTION

How does a student know when they have produced an invalid or nonsense answer to a physics problem? The simplistic answer is that students need to engage in "sense making." However, at the upper-division level, sense making is not a simple either-or proposition. During the course of solving a problem, students have multiple opportunities to engage in sense making at many different levels.

In this project, we looked at 17 juniors in Oregon State University's Paradigms in Physics program.[1,2] Students were working in class in five different groups of two or three using large white boards while attempting to find the magnetic vector potential for a spinning ring of charge. Students had already completed the first 21-contact-hour course in the Paradigms sequence, Symmetries, and were currently enrolled in the second course, Vector Fields.[3]

None of the students reached a correct solution in the time allotted, but we found a large difference in the degree to which students engaged in sense-making strategies. Most of the students engaged in some forms of sense making, but were frequently willing to accept errant results. We will employ Sayre and Wittmann's[4] concept of "resource plasticity" to shed light on student thinking, problem solving and sense making.

The first section of this paper defines the concept of "plastic" vs "solid" resources and sets the context for the research. In a larger study,[5] we found many examples of students using varying degrees of plastic resources in a variety of ways. However, for this paper we consider only two contrasting examples of student interactions that highlight the concept. Finally, we consider the instructional implications of these findings.

BACKGROUND AND CONTEXT

When looking at student learning in physics, Hammer[6] proposed using an analogy from the language of computer programmers. "Resources" in computer programming refer to chunks of code that are taken unaltered and can be transferred as a single piece to a new situation, without needing to think about any of its sub-pieces. Since then, a resources perspective has become a staple of physics education research.

Sayre and Wittmann subsequently modified the definition of "resources" to consider a continuum of understanding and subsequently applied this new definition in the context of upper-division physics. Specifically they consider the degree to which student resources are "solid" versus "plastic." Solid resources tend to be older, readily available, easy to use, well consolidated and well connected to other resources. Plastic resources tend to require more effort to use, are open to re-evaluation and are often reliant on justifications from more solid resources.

Sayre and Wittmann applied the concept of resource plasticity when looking at students choosing which coordinate system (Cartesian vs. polar) to use when solving for the time required for a pendulum to swing over a given arc. They conclude that plasticity in understanding the coordinate system strongly influences which coordinate system students choose to use.

In this study, finding the magnetic vector potential for a spinning ring of charge was the third in a series of four ring problems given to the students over the course of a few weeks. The previous problems involved finding the electric potential and electric field around a stationary ring of charge. Therefore, the students had recent experience with the geometry of the problem, up to and including evaluation of the denominator of the integral, but not with finding the current in the numerator. So their sense making in this episode is focused on understanding current.

As a prompt for this particular problem, the instructor grabs a hula hoop, holds it up, and tells students the following: "We're going to go back to the case of the ring. We have a ring with total charge Q, radius R, and now we're going to make it spin so that the charge is moving. So you have a spinning ring of charge with period capital T, and I want you to write an expression for the magnetic vector potential anywhere in space in a way that Maple could evaluate it."

The students are given the general equation for the magnetic vector potential due to a volume current $\vec{J}(\vec{r}\,')$

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi}\iiint \frac{\vec{J}(\vec{r}\,')d\tau'}{|\vec{r}-\vec{r}\,'|} \qquad (1)$$

(where $\vec{r}$ denotes the position in space at which the magnetic vector potential is measured and $\vec{r}\,'$ denotes the position of the current segment), but were expected to generalize this expression to a line current. Ideally, students would eventually reach a solution in the form

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi}\frac{QR}{T}\int_0^{2\pi} \frac{(-\sin\phi'\,\hat{\mathbf{i}} + \cos\phi'\hat{\mathbf{j}})\,d\phi'}{\sqrt{r^2 - 2rR\cos(\phi-\phi') + R^2 + z^2}} \qquad (2)$$

Among the things students had to consider were: the velocity of the rotating ring, the charge density, the magnitude of the current, the direction of the current, reducing the general formula down to one dimension, figuring out how to "chop and add" to set up the integral, expressing $\vec{r}-\vec{r}\,'$ in cylindrical coordinates, and expressing $\hat{\phi}'$ in rectangular coordinates.

There were certain components of the ring problem that students could handle quickly and easily by relying only on solid resources. For example, all students seemed to be able to apply, without any noticeable effort, that the ring circumference is $C = 2\pi R$ and that the charge density is $\lambda = Q/\ell$.

On the other end of the spectrum, by analyzing the data of students solving the problem, we found two aspects of the problem that appeared novel to all students. One was integrating while having current as a vector in the integrand. Students had to figure out how to handle the fact that the direction of the current was not constant during integration. As a consequence, they encountered the second novel aspect of the problem, how to express the current direction $\hat{\phi}'$ in terms of Cartesian basis vectors in the form of $-\sin\phi'\,\hat{\mathbf{i}} + \cos\phi'\hat{\mathbf{j}}$.

Between the extremes of unfamiliar concepts and mastered concepts lay substantial middle ground. When students attempted to solve aspects of the problem that were unfamiliar, there was a wide variation in the degree to which students employed plastic resources and solid resources. In several cases, students employed plastic resources without explicitly considering that their understanding was incomplete.

EMPLOYING PLASTIC RESOURCES

In the following case, students appear to be unaware that they do not have a firm grasp on the concept they are trying to use. These three students are trying to determine the direction of the magnetic vector potential, and one student proposes (incorrectly) that the right-hand rule applies. All the members of the group rapidly agree.

Group 4, which consists of Stan, Robert and Kevin, draw a picture of the ring and are currently adding to this drawing. The following is an outtake of their dialog:

Stan: "Spinning..." [draws a curved arrow next to the ring],

Robert, "Draw"

Stan, "...current..." [draws an upward vector along *z*-axis],

Robert, "Yeah."

Kevin: (talking over Stan) "There's got to be some moment of inertia in here."

Stan, "...right hand rule..." [gestures fingers curled, thumb up] "...or, *B*..." [Stan labels vertical arrow "*B*"] "...or *A*..." [changes "*B*" to an *A* (Fig. 1)]

Robert [repeatedly gestures curled fingers with thumb up] "Basically,… basically the field is going to go up...the whole right-hand-rule thing...spins that way, current up.

Stan gestures curled fingers with thumb up

Kevin draws a new, larger ring

Robert [referring to ring]: "Well yeah. So if you say it's spinning that way..." [draws arrow on ring]

Stan: "Then, then it'll be up." [labels upward on the *z*-axis *A*(*r*)] "*A*(*r*)"

Kevin: "Yeah."

Robert: "Yeah."

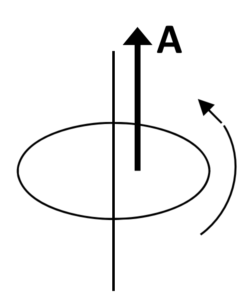

FIGURE 1. Stan labels vertical arrow "A"

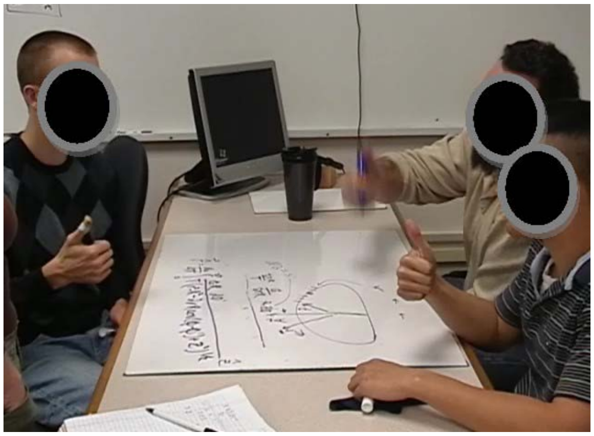

FIGURE 2. The members of group 4 are all gesturing the right-hand rule when trying to determine the direction of magnetic vector potential.

The students in Group 4 all incorrectly agreed that the “right-hand rule” applied to finding the direction of the magnetic vector potential. Furthermore, members of the group express different interpretations of what goes “up” using the right-hand rule, including current, magnetic field, and magnetic vector potential. By using incomplete or “plastic” understanding of magnetic vector potential, magnetic field, and the right-hand rule they settled on what appeared to be an “easy” solution to finding the direction. This prevented them from realizing the challenging aspects of finding the direction of magnetic vector potential.

EMPLOYING SOLID RESOURCES

In contrast, the following example shows a different group, Group 5, with Shawn, Biff, and Devin, who are also trying to establish the direction of the magnetic vector potential. In group 5, Biff proposes finding the direction of the vector potential using the right-hand rule. However, in this case, Shawn challenges its utility. Shawn doesn’t actually directly challenge that the right-hand rule applies, but instead considers that for a point not on the z-axis, the right-hand rule would not yield a trivial answer.

Biff, "Say by the right hand rule, it's in this direction." [gestures right hand rule with thumb up]

Shawn, "But if you're, like, way up here at some weird point..." [points to a place on board away from the ring and off-axis]

Devin, "Yeah, but right-hand rule is kind of a sketch. You still have to have an exact [inaudible].”

Shawn, "Like if you're way up here," [draws an external point (Fig. 3)], "like, which," [gestures from ring to external point] "I mean which way is it going to point?" (Shawn points back and forth between different locations on the ring and the external point).

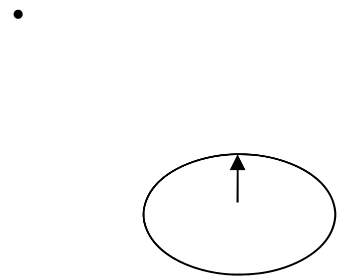

FIGURE 3. Shawn draws an external point.

Shawn initially accepts his partner’s suggestion to use the right-hand rule, but quickly shows that it would not easily show the direction of the magnetic vector potential at “some weird point”. Unlike Group 4, in which all the members settled on an incorrect solution that relied on their incomplete understanding, Shawn connects to something he geometrically understands, which is that the “right-hand rule” does not provide for a trivial solution at points that are not on the central axis. This prevents Shawn from accepting an incorrect solution. Although Shawn never reaches a correct solution for the direction of the magnetic vector potential in the allotted time, he also has engaged in sufficient sense making to realize that he is not yet a point where he can produce a solution.

DISCUSSION

The students in Group 4 had limited, if any, valid understanding of magnetic vector potential and limited understanding of the right-hand rule. Their use of the right-hand rule fit the description of a highly plastic resource; it was recently acquired and not well consolidated nor well-connected to other resources. When these students attempted to build new understanding of magnetic vector potential based on attempting to apply a highly plastic resource, they ended up producing and accepting incorrect results.

Shawn’s understanding of the right-hand rule was still not entirely solid, because he was at least willing to entertain the (incorrect) idea that it was directly applicable to finding the direction of magnetic vector potential. However, his understanding of the right-hand rule showed more qualities of a solid resource, such as being actively connected to the concept of trying points other than “special points” (in this case trying an off-axis point). When Shawn, and students like him, attempted to use geometric reasoning to connect to solid understanding, they did not settle on errant results.

Most of the 17 students studied fell between the students who employed consistent, strong sense-making strategies, and those who employed none.

Many students made some attempt to interpret and understand their results, but were often willing to accept weak or partial understanding. Only a few students consistently insisted on a much firmer footing for their sense-making.

IMPLICATIONS

These data suggest that many students are willing to accept incorrect results as valid, and that those who tend to connect new understanding to resources that are more solid do not. If we want students to engage in a high degree of sense making at the upper division level, it may be important to help students learn to connect to more solid resources and not accept highly incomplete understanding as "good enough".

One conjecture is that time spent allowing to students to build firmer connections to prior learning may have longer term advantages. When students have a well-connected solid resource, such as the formula for the circumference of a circle, they are often able to use this resource quickly and with a minimum of effort. When students are attempting to utilize a more plastic resource, they are likely to need longer times in order to solidify that resource and connect it to other more solid resources. Students may need to be reminded of relationships or be given additional time to reconstruct them.

Instructors may need to build opportunities to do this into the curriculum. While expediency may result in some topics needing to be addressed with a "survey" or "tour bus" approach, at other times students could benefit from building deeper understandings, and instructors may want to create the time for students to do this. Dividing topics into those for which "survey-level" understanding is expected and those for which "deep" understanding is expected would be one way to reallocate time instead of a more equal division of time among topics.

A second conjecture is that if instructors require (*i.e.* require in the sense of keeping an interaction going in the classroom until students do it) students to sometimes work their way deeper and deeper until they reach solid resources, then students may come to realize what that feels like and start looking for it in more of their reasoning. Active engagement, such as group problem solving, could be one environment in which this could occur. Through repeated probing of students thinking with, "How do you know this?" or "Justify this claim," students may come to expect deeper levels of understanding when problem solving.

For the Paradigms students, of course, this was not the end of the problem. Their problems evaluating the numerator of the integral in Eqn. (1) was an expected design feature of the sequence of activities[3,7] which was addressed in a follow-up whole-class discussion. Subsequently, the class uses *Maple* or *Mathematica* to create a visualization. Several days later, students were given the opportunity to work in groups to find the magnetic field due to the same spinning ring, which provided an opportunity to revisit these concepts. This data has yet to be analyzed to see if students show significant improvements when solving the subsequent problem.

The main author of this paper (LTC), currently teaching at the high-school level has found (anecdotally) that implementing strategies suggested by this research apparently leads to substantial improvements of depth of understanding and overall knowledge. The teacher picks two topics from the course and insists that students understand those well. The teacher requires each student to articulate and apply the underlying principles in a variety of contexts. Time to do this was created by covering other topics in less depth. Whether this translates into students using deeper sense-making strategies in new or unrelated contexts is a subject for future research.

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